Problems with Omniscience

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I. Introduction: 'God' and 'Omniscience'

Here are my ground rules:

'God' is defined as a 'a being necessarily existent, timeless, ubiquitous, omnipotent, omniscient, and morally perfect, the creator of the universe.' That is the target. If no being fits the bill, there is no God.

I am not interested in 'God' used as a purported name for something-big-but-we-know-not-what, allowing claimants to change their claims regarding any or all purported properties. In my usage, we could not find out that God is very knowledgeable but not omniscient, that he is occasionally cruel, or that he is actually a fairly normal mortal selling insurance in the suburbs of Chicago. The properties of omnipotence, omniscience, and moral perfection do not merely characterize God contingently. They define the meaning of the term 'God' on a central classical conception. That is the target. If no being fits the bill, there is no God.

The standard properties listed face logical problems both singly and in combination. I will concentrate on just one of those properties, and in isolation: the notion of omniscience, or of being all knowing. I focus on the classical definition: a being is omniscient if it knows everything. More precisely, a being is omniscient if and only if it knows all truths—and, we
have to add, believes no falsehoods.\textsuperscript{1} The beliefs of an omniscient being would correspond precisely to a set of all and only truths.

Here again let me emphasize the ground rules. Omniscience demands knowing everything. It is not satisfied by knowing all of some limited class of propositions. It is not satisfied by knowing everything that a being of some particular type might be able to know, for example. It is not satisfied by some we-know-not-what optimum of knowledge that falls short of knowing everything. If no being knows literally everything, no being is omniscient. If no being is omniscient, there is no God.

\textbf{II. The Divine Liar}

Consider the following:

1. X does not believe that (1) is true.

For X we substitute any name or referring expression. For any such substitution we can ask whether (1) is true or false. If (1) is true, X does not believe it, and thus X cannot be said to know all truths. If X is false, it is false that X does not believe it. It must then be true that X believes it. X therefore believes a falsehood.

The trap is set. In light of (1), there is no X that can qualify as omniscient. For any candidate put forward, there will be a form of (1) that must be either true or false. But if true, the candidate cannot qualify as omniscient. If false, the candidate cannot qualify as omniscient.

There can be no omniscient being, and thus can be no God.

\textbf{III. Omniscience and the Knower}

It is clear that the problem above is constructed on the model of the Liar. A second problem for omniscience follows the paradox of the Knower.

Consider any formal system containing axioms adequate for arithmetic. Those axioms formalize statements as simple as the principle that every natural number has a successor, that zero is the successor of no natural number, and that addition and multiplication operate in the familiar way. We take such axioms to be true on their face. We take them to be true, moreover, because we take arithmetic to be true.

For any such system it is well known that we will be able to encode formulae recoverably as numbers. We will use A to refer to the numbered encoding for a formula A. It is well known that for any such system we will be able to define a derivability relation I such that \( \vdash I(A, B) \) just in case B is derivable from A.

Let us introduce a symbol ‘\( \nabla \)' within such a system, applicable to numerical encodings \( \bar{A} \) for formulae A. We might introduce ‘\( \nabla \)' as a way of representing universal knowledge, for example—the knowledge of an omniscient being within at least the realm of this limited formal system. Given any such symbol with any such use we would clearly want to maintain each of the following:

- If something is known by such a being, it is so \( \nabla(\bar{A}) \rightarrow A \)
- This fact is itself known by such a being. \( \nabla(\nabla(\bar{A}) \rightarrow A) \)
- If B is derivable from A in the system, and A is known by such a being,
- B is known by such a being as well. \( I(\bar{A}, \bar{B}) \rightarrow \nabla(\bar{A}) \rightarrow \nabla(\bar{B}) \)

The simple truth, however, well established as a logical theorem, is that no symbol can consistently mean what we have proposed ‘\( \nabla \)' to mean, even in a context as limited as formal
systems of arithmetic.\(^2\) The addition of these simple claims to any system containing basic truths of arithmetic would give us a contradiction.

Given the basic truths of arithmetic, omniscience is a contradictory concept. Because arithmetic is true, there is no omniscient being. Because arithmetic is necessarily true, no omniscient being is possible.

**IV. The Escape Route of Hierarchy**

Both of the problems raised to this point—both the Knower and the Divine Liar—can be finessed by familiar hierarchical techniques. The proposal in Russell, in Tarski, in Kripke and in others is that truth and related predicates form a hierarchy of different levels, each of which applies only to statements (including statements involving truth) on lower levels. On such an approach we can say that we know precisely what goes wrong with the Divine Liar and '\(\forall\)': each attempts to apply a truth-related predicate beyond its hierarchically restricted reach.\(^3\)

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\(^3\) Nor will it suffice to envisage an omniscience operator in place of an omniscience predicate. See Grim, "Operators in the Paradox of the Knower," *Synthese* 94 (1993), 409-428.

The formal charms of a hierarchical approach are many. But one thing such a route does not offer is any hope for the concept of omniscience. If truth forms a stratified hierarchy, there can be no notion that applies to truths on *all* levels. That is precisely the point of the appeal to hierarchy; were one to quantify over all levels, or to reinstitute a notion of 'truth at any level,' all the machinery would be in place to create the same logical problems in the same form. If hierarchy prohibits any notion of 'truth on all levels,' however, it also prohibits any notion of omniscience; knowledge of truth 'on all levels' is precisely what the concept of omniscience would demand.

The appeal to hierarchy, though of logical interest and importance, offers no solution for the logical difficulties of omniscience.

**V. The Cantorian Argument**

The beliefs of an omniscient being, we have said, would correspond precisely to a set of all and only true propositions. Because there can be no set of all and only true propositions, there cannot be what an omniscient being would have to know, and thus there can be no omniscient being.

The proof that there can be no set of all truths goes as follows. Consider any set of truths $T$:

$$T = \{t_1, t_2, t_3, \ldots \}$$

And consider the elements of its power set $PT$, containing all subsets of $T$:

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4 Despite the standard linear progression, there is no suggestion here that any such set be denumerable. The argument is generalizable to a 'set of all truths' of any size.
To each element of the power set there will correspond a unique truth—at least the truth that that element contains a particular truth $t_1$ as a member, for example, or that it does not contain $t_1$ as a member:

$$t_1 \in \{ t_1, t_2, t_3 \}$$
$$t_1 \notin \{ t_1, t_2 \}$$

By Cantor's theorem, we know that the power set of any set contains more members — it is larger, or has a greater cardinality — than the set itself. There will then be more truths than are contained in $T$. But $T$ can be taken as any set of truths. For any set of truths, we can show that there are more truths than it contains. There can therefore be no set of all truths.

The Cantorian argument strikes at a crucial assumption essential to any notion of omniscience—that truth approaches some final totality or 'all.' That assumption is provably false. Truth explodes beyond any attempt to capture it as a totality, and that explosion takes the form of logical contradiction. There can therefore be no coherent notion of omniscience.
VI. The Cantorian Argument Extended

Is there no escape from the Cantorian argument? One might propose that truths form a 'proper class' instead of a set, where important restrictions are placed on 'classes'. In a range of formal theories classes are introduced in such a way that we are prohibited from forming classes of classes in certain ways, thereby restricting the reach of the power set axiom. But those maneuvers are widely recognized as both logically ad hoc and philosophically unsatisfying: some truth while undoubtedly hold regarding any bunch, cluster, or collection of truths, raising precisely the same problems as before.

A number of authors have suggested that the real target of the argument is not truths in their plurality but the notion of truth as a 'completed totality' in the singular. What if we think of truth as an inescapable 'many' rather than a 'one'? Richard Cartwright, D. A. Martin, Keith Simmons, and Alvin Plantinga have all made proposals along these lines.

5 Gary Mar proposes Quine's New Foundations NF as a class theory in which Cantor's theorem and thus the Cantorian argument will not hold ("Why 'Cantorian' Arguments against the Existence of God Do Not Work," International Philosophical Quarterly 33 (1993), 429-499). But the technical disappointments of NF are well known (see for example J.B. Rosser and Hao Wang, 1950, "Non-standard Models for Formal Logic," Journal of Symbolic Logic 15 (1950), 113-129). Of greater philosophical importance in the present context is the fact, detailed below, that the argument can be phrased entirely in terms of intuitive principles regarding truth, without sets or classes.

Although tempting, the appeal to truth as a 'many' that cannot form a 'one' turns out to be ineffective, because the core argument does not in any essential way on reference to a single set, class, or collection of all truths. It can be phrased directly in terms of a 'many,' entirely in the plural, and using only a notion of relations between things.\(^7\)

For properties \(P_1\) and \(P_2\), the required formal properties of relations can be outlined as follows. Those things that are \(P_1\) can be mapped one-to-one into those things that are \(P_2\) just in case there is a relation \(R\) such that:

\[
\forall x \forall y[\exists z(P_2z \land Rxz \land Ryz) \rightarrow x = y] \land \forall x[\exists y(\forall z(P_2z \land Rxz \leftrightarrow z = y))]
\]

Those things that are \(P_1\) can be mapped one-to-one onto those things which are \(P_2\) just in case (here we simply add a conjunct):

\[
\forall x \forall y[\exists z(P_2z \land Rxz \land Ryz) \rightarrow x = y] \land \forall x[\exists y(\forall z(P_2z \land Rxz \leftrightarrow z = y))]
\]

\[\land \forall y[\exists x(P_1x \land Rxy)]\]

It will be true for some \(P_1\) and \(P_2\) that a mapping into is possible but a mapping onto is not; relative to the things that are \(P_1\) there will be too many things that are \(P_2\) to allow a full mapping of those things that are \(P_1\) onto those things that are \(P_2\).

Consider, then, any 'many' truths you like—the truths that any particular being knows, for example. Consider also truths about one or more of those truths. Using the notions outlined above, phrased entirely in the plural, it is possible to show that the first truths can be mapped into but not onto the second truths. There are too many of the latter. No matter however many truths are at issue, therefore, they cannot be all the truths. Appeal to a plural 'many' rather than a single set- or class-like 'one' offers no escape; it is still true that there can be no omniscient being.

\(^7\) It is true that relations are often instantiated in set theory in terms of ordered sets. But we can clearly think in terms of properties and relations independently of set-theoretic extensions.
VII. The Option of New Logics

Given elementary logic, the inescapable conclusion is that omniscience is impossible. Any God would have to be omniscient. Given elementary logic, there can therefore be no God.

One way of phrasing the result is this: Within any logic we have, there can be no totality of truths. Within any logic we have, there can therefore be no omniscience. I think the qualifier 'within any logic we have' deserves to be taken seriously. But it is not the simple escape route it might appear.

'Ah, within any logic we have... But perhaps within some new logic there will be a place for a totality of truths, and will be a place for omniscience...' The problem with such a response is that it may be as empty as it is easy. In the face of any argument, however tight, in defense of any inconsistent concept, however clear the contradictions, one might always say 'Ah, within any logic we have... But perhaps within some new logic...' At this point in the discussion, therefore, such an appeal no more constitutes a genuine reply than does the same appeal in defense of square circles, married bachelors, or army intelligence. Within any logic we have, there can be no married bachelors. Within any logic we have, there can be no omniscient being.

What gives the invocation of new logics some greater bite in this case is that some initial steps have been taken in such a direction. In recent work, Nicholas Rescher and I have explored options for alternative logics regarding indeterminate collectivities.\(^8\) Such collectivities would include semantically indeterminate collectivities such as 'the world's foothills,' indeterminate in membership because of the vagueness of the term 'foothills.' They would also include

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epistemically indeterminate collectivities such 'people no-one knows ever existed.' It is obvious that there are such people, but it is equally obvious we cannot specify them. Indeterminate collectivities will also include ontologically indeterminate collectivities such as the electrons in a particular lump of Californium that will decay over the next minute. What is important for our purposes is the idea that indeterminate collectivities might include logically indeterminate collectivities as well.

Among logically indeterminate collectivities, we have proposed, might be 'plena,' envisaged explicitly as hyper-cardinal multiplicities with 'number beyond number.' The idea is to recognize plena as collectivities such that every sub-collectivity \( s \) of a plenum \( P \) stands in unique correlation with some member of \( P \) itself. Using \( \exists_x \) to stand for 'exists in unique correlation with \( s \),' the definitional condition for plena \( P \) is then

\[
(\forall s)(s \subset P \rightarrow (\exists_x) x \in P)
\]

So conceived, there would be a mapping of all sub-collectivities of a plenum into its members, that is, a mapping \( m \) such that

\[
(\forall s)(s \subset P \rightarrow (\exists x)(x \in P & x = m(s) \land (\forall s')(s' \neq s \rightarrow m(s') \neq m(s)))
\]

If any sense is to be made of propositions, facts, or truths in full collective generality, it would have to be as plena. In full collective generality these would boast a characteristic that no set and no class can possibly satisfy—the condition of having as many elements as their power collectivities, precisely the definitional characterisation of plena. For each sub-collectivity of a plena of truths, for example, there will be a unique truth, and thus 'more' elements of the plena than are evident at any point of comprehension. Plena would be precisely those multiplicities that refuse to cohere in some final totality, a completed 'whole' or 'all.'
What 'new logic' would be required to make room for plena and other indeterminate collectivities? There appear to be three possibilities.  

One alternative is to abandon the Law of Excluded Middle, which plays a major role in all of the arguments above. One would have to recognize propositions that are neither true or false, and would lose standard patterns of inference in consequence—the inference from the falsity of 'X does not exist' to the truth of 'X exists,' for example. What is required is a *strong* abandonment of the Law of Excluded Middle, not to be satisfied simply by the addition of a further—'neutral,' perhaps, for 'neither true nor false.' What is demanded is rather a logic in which there are semantic black holes: propositions with no semantic value at all, with the emphatic reminder that 'having no semantic value' is not itself to be treated as a semantic value. Within such a logic, it turns out, negation could no longer be exhaustive: one could not use 'not P' to target *all* the cases in which P does not hold, for example. 

It is an indication of how drastic this first alternative is that a second alternative is no more drastic: the alternative of abandoning the Law of Contradiction instead. On this second approach, there would be propositions that are *both* true and false. Proof of the truth of a proposition would therefore not entail that it might not be false as well. Proof of the truth of a theological claim would not guarantee that it might not be false nonetheless. In such a logic the notion of an exclusive negation would lose its grip: 'not P' would no longer exclude P. 

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9 Here I cannot go into detail. The interested reader is referred to chapter 6 of Rescher and Grim, *Beyond Sets*, op. cit. 

10 In classical logic, of course, the two are equivalent: \( \forall p (p \lor \neg p) \leftrightarrow \exists p (p \land \neg p) \). In alternative logics that equivalence will not generally hold, but abandonment of either demands major logical sacrifice.
Those alternatives are each so drastic, it turns out, that a more comfortable logic may be one in which both the Law of Excluded Middle and the Law of Contradiction fail to hold in full generality. In such a there would be no univocal concept of negation, replaced instead with two negations—one exclusive but not exhaustive, the other exhaustive but not exclusive.

'Ah, within any logic we have omniscience proves impossible ... but perhaps within some new logic...' As argued elsewhere, each of the deviant logics mentioned is worthy of further work, both formal and philosophical. But even first steps make it clear that any such logic invokes a realm far stranger than might at first appear. The concept of omniscience, one might argue, is firmly embedded in a classical conception of the fabric of the universe that is woven using classical logic throughout. To try to save a concept of omniscience by abandoning the logic constitutive of its natural context would be far more destructive, entailing far more radical revisions even in theology, than to simply abandon the concept itself.

There is of course a further and far less drastic alternative. If truth and related predicates form a structured hierarchy, the Cantorian argument will lose traction much as did the Knower and the Divine Liar. But structured hierarchy would come at the same cost here as before, essentially conceding the conclusion of the Cantorian argument. Given such a hierarchy there will then be no concept that applies to all truths, and correspondingly no coherent notion of omniscience.

VIII. The Problem of the Essential Indexical

11 See Rescher and Grim, Beyond Sets, op. cit., esp. chapter 6.

12 Prospects for other forms of hierarchy, no less surprising and perhaps not entirely distinct from abandonment of the Law of Excluded Middle or Law of Contradiction, are discussed in Rescher and Grim, loc. cit.
Omniscience faces further logical difficulties, distinct from those above. Primary among these is the problem of the essential indexical.

Consider a case borrowed from John Perry. I follow a trail of spilled sugar around and around a tall aisle in the supermarket, in search of the shopper who is making a mess. Suddenly I realize that the trail of sugar is spilling from a torn sack in my cart, and that I am the culprit—I am making a mess.

What it is that I realize at that point is that

2. I am making a mess.

The interesting point is that this proposition is not the same as

3. Patrick Grim is making a mess,

nor can it be the same proposition as

4. He is making a mess.

where I am the "he" that is indicated.

We can easily construct cases in which I know (2) or (3) without knowing (1). In an amnesia case I may know that Patrick Grim is making a mess without realizing that I am Patrick Grim, for example, and thus without knowing (1). I may see that he is making a mess—that oaf in the fish-eye mirror—without yet realizing that oaf is me. What I know in knowing (2) or (3) therefore falls short of what I know in knowing (1). The proposition expressed by (1) will not be adequately captured by anything like (2) or (3).

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Another clear indication that (2) and (3) express propositions different from (1) is that (1) offers a complete explanation for things that (2) and (3) cannot. When I stop myself short in the supermarket, gather up my broken sack, and start to tidy up, my doing so may be quite fully explained by saying that I have realized that I am making a mess. But it could not be fully explained by saying that I realize either (2) or (3). In (1) 'I' plays is an essential indexical—essential to what I know in knowing that I am making a mess. In order for (2) or (3) to offer a full explanation for my behavior, I at least have to add the knowledge that I am Patrick Grim, or that I am him—thereby reintroducing the essential indexical 'I'.

The argument can be also be constructed for indexicals other than 'I'. What I come to realize when I realize that the meeting is starting now is not simply what others know when they know that the meeting starts at noon, expressed timelessly and without tense. The fact that I know the meeting is starting now fully explains my hurry to gather up the required materials in my office. My knowing that the meeting starts at noon would not explain that hurry...unless of course we added that I also know that it is now noon, thereby reintroducing the temporal indexical. My knowing what I know now explains something that my knowing what others might know timelessly or at other times could not explain. The two must therefore be different: what I know now is not merely what they know then. It follows that there are things known now that simply cannot be known timelessly.14 We can run a similar argument using indexicals of place regarding my knowledge that the bomb is to fall here.

But let us return to the indexical 'I'. The closest others can come to knowing what I

14 It follows that no timeless being can be omniscient. Because that is a combinatorial problem for multiple attributes I put it aside here.
know in knowing (1) is knowing something like (2) or (3). But neither of these amounts to what I know in knowing (1). There are therefore things that I know—what I know when I know I am making a mess, for example—that no other being can know.

An omniscient being would know everything—certainly everything known by any being. But no being other than me can know some of the things that I know, and I am clearly not omniscient. There can therefore be no omniscient being.

**IX. Indexical Escape Routes**

What options are open for the defender of omniscience in the face of the argument from essential indexicals? As indicated in the ground rules, I am not interested in any evasive or deflationary re-definition of 'omniscience' as less than 'omni,' such that 'knowing everything' could be satisfied by knowing significantly less.  

15 Yujin Nagasawa attempts precisely this type of definitional evasion: "[E]ven if Grim is right in saying that God cannot know what I know," he claims, "there is no reason to accept his conclusion that God cannot be omniscient." Nagasawa's strategy, following the tracks of a similar strategy regarding omnipotence, is to redefine the term such that a being that knows all that a being of that kind can essentially know would qualify as omniscient. I object to any move toward redefinition at all. But it's also true that on such an approach 'omniscience' would come dirt cheap. A stone is essentially incapable of knowing anything. Were omniscience to require of a being knowing merely all that a being of that type could essentially know, any stone would qualify as omniscient. On such a redefinition, therefore, there would literally be as many omniscient beings as there are grains of sand on the beach. See Yujin Nagasawa, "Divine Omniscience and Knowledge *de se,*" *International Journal for Philosophy of Religion* 53 (2003), 73-82.
The only direct attack is to insist that another *could* know what I know in knowing that I am making a mess. There are indeed precedents for such a move in other work on indexicals. Consider, for example, the case in which I see the mess-maker in the fish-eye mirror at the end of the supermarket aisle and come to the conclusion that *he* is making the mess. My further realization a moment later that it is I who am making a mess, it has been proposed, involves not a new proposition but a change of perspective. What differs, John Perry proposes, is not what is known but simply the 'belief state' in which it is known.\(^{16}\)

On such an approach, omniscience might again seem possible. But what it would demand would ultimately be a notion of knowledge radically different than ours. When I suddenly realize that the man in the mirror is *me* there is clearly something that I have learned. There is something that I didn't know before that I do know now. There is indeed a piece of crucial information that I have just acquired that I did not have before — the fact that it is *me* that is making a mess. All of these reflect our standard concept of knowledge. The simple fact that there *is* something that I learn, recognize, or come to realize in such a case is a fact to which no 'belief state' account is capable of doing justice. On any such account it is emphatically *not* the case that there would be anything new to learn.

Similar comments apply to other indexicals. When it dawns on me not merely that that the meeting starts at noon, but that it starts *now*, there is something that I come to realize for the first time. When I knew merely that the bomb was going to fall at a spot marked on the map there was a crucial piece of information I lacked. What I didn't yet realize was that it was going

We can also appeal to the fact that what I know carries over to other propositional attitudes. What I come to realize in the mess-making case is precisely the thing that I am then ashamed of—the fact that it is *me* that is making a mess. What I might not have believed before, what suddenly surprises me, and what I realize with a shock is that the meeting is starting *now*. What I come to know about the bomb's fall is precisely what I was afraid of—that it would fall *here*.

Given our concept of knowledge, and given the links of that concept with other propositional attitudes, it is not merely a 'belief state' or a perspective that changes when I come to realize, for example, that it is *me* that is making a mess. At least part of what has changed is what I know. In the straightforward and familiar ontology of what is known, the argument from indexicals does show quite explicitly that no being can know what I know.

Omniscience is defined in terms of knowledge, taken in the standard and familiar sense. In that standard and familiar sense there can be no being that knows everything. There can be no omniscient being.

**X. Against Omniscience**

Logical problems regarding the concept of omniscience seem to me as close to a knock-down argument as one ever gets. Given our concept of knowledge and its objects, no being can know everything known; there can be no omniscient being. Within any logic we have, there can be no totality of truths. Omniscience is defined in terms of a totality of truths. There can thus be no omniscient being.

One might propose changing logics. One might propose changing concepts. But one might *always* attempt *either* of those evasive maneuvers, in the face of *any* argument, however
tight, for any conclusion, however solid. The logical problems facing omniscience seem to me as close as to a knock-down argument as one ever gets.

There may be good reasons to explore alternative logics. But to abandon fundamental principles of classical logic in order to save a concept soaked through and through with the classical logic of classical theology would not seem to be among them. To change our concept of knowledge in order to save a concept of omniscience would seem futile; it would no longer be the standard concept of omniscience one would be saving.

Works Cited


_________, *Beyond Sets: Toward a Theory of Collectivities*, Ontos/Verlag, 2011.


**Recommended reading:**


