The Being That Knew Too Much

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1. Introduction

John Abbruzzese has recently attempted a defense of omniscience against a series of my attacks.1 This affords me a welcome occasion to clarify some of the arguments, to pursue some neglected subtleties, and to re-think some important complications. In the end, however, I must insist that at least three of four crucial arguments really do show an omniscient being to be impossible. Abbruzzese sometimes misunderstands the forms of the argument themselves, and quite generally misunderstands their force.

The fourth argument—the Cantorian argument against omniscience—has a more complicated status. The possibility that such an argument is somehow self-defeating is an intriguing one, with deep philosophical implications for propositional quantification in general.2 This is the one aspect of my earlier work about which I have come to have the gravest second thoughts. But the second thoughts I have to offer here are second thoughts about the nature of a particular class of arguments and perhaps about the character of argument in general. I don’t believe those second thoughts ultimately offer any hope for omniscience.
2. The Essential Indexical Argument

No one else knows what I know in knowing that:

1. I am making a mess.

The closest others may get is knowing that

2. Patrick Grim is making a mess,

or perhaps

3. He (indicating me de re) is making a mess.

But what they know when they know (2) or (3) is not what I know in knowing (1).

It is easy to construct cases which make it clear that I might know what others know in knowing (2) without knowing what I know in knowing (1). I might know that Patrick Grim is making a mess without knowing that I am making a mess, for example, simply because I fail to realize that I am Patrick Grim.

It is also easy to construct cases with fish-eye mirrors which make it clear that I might know what others know in knowing (3) without knowing what I know in knowing (1). I might know that he is making a mess without knowing that I am making a mess simply because I fail to
realize that he that clumsy oaf in the mirror is me.

The argument can be constructed for indexicals other than \( I = \) as well. What I come to realize when I realize the meeting is starting now is not simply what others know when they know that the meeting starts (timelessly) at noon. The fact that I know the meeting is starting now fully explains my hurry to gather up materials required for the meeting. My knowing that the meeting starts at noon would not explain that hurry, unless we added that I also knew that it is now noon, thereby reintroducing the indexical. What I know now explains something that my knowing what others might know timelessly or at other times could not explain. The two things must therefore be different: what I know now is not merely what they know then. A similar argument using indexicals of place can be rehearsed regarding my knowledge that the bomb is scheduled to fall here.

It thus appears that there are things that can be known now that simply cannot be known timelessly. No timeless being can therefore be omniscient. It appears there are things that I know when I know I am making a mess, for example that others simply cannot know. If so, since I am not omniscient, there are things that I know that no omniscient being could know. But an omniscient being would have to know at least all that I know. There can therefore be no omniscient being.

Abbruzzese’s defense of omniscience in such cases amounts to the observation that A. feelings of any kind, I would think, are not included in what we would call knowledge.

\[ \ldots \text{what this knowledge does not contain is the feelings of guilt or embarrassment I experienced.} \ldots \]
In light of this, there appears to be an easier way to resolve the problem of essential indexicals and the omniscience of the MPB [Most Perfect Being]. By definition, the nature of the MPB's knowledge is perfect and would contain, as Grim should admit, all facts. But as we have seen, the knowledge that I have . . . comprises only factual, not emotional parts. (28)

This seems to be a simple misunderstanding of the character of the argument from essential indexicals, not only in my treatment but in Castañeda's, David Lewis's, John Perry's, or Steven Boër and William Lycan's. None of the arguments above suggest that my feelings about anything—my mess-making, the meeting starting, or the bomb dropping—are part of what I know. The structure of the argument would be the same for cases in which I had no particular feeling at all about what was at issue—cases in which I realized that I am the man wearing a purple tie (ho hum), or that it is now 1:00, or that it is here that I bought a paper a week ago.

The argument can be presented by calling attention to certain feelings explained by what I know. Beyond that, feelings simply have nothing to do with it. What the argument shows is that two pieces of knowledge cannot be the same because (1) I can know one thing without knowing the other, or (2) my having one explains things that my having the other could not. We don't need feelings to go on to argue that these two pieces of knowledge cannot be the same. The non-identity of discernibles will suffice.

Although entirely different than Abbruzzese's defense, it should be noted that there are some proposals in the later work on essential indexicals that might appear to offer a loophole for omniscience. Both John Perry, and Steven Boër and William Lycan, suggest that what I know
when I know that I am making a mess, and what others know when they know that *he* is making a mess, *is* after all the same thing known. What differs, Perry proposes, is not what is known but simply the *belief state* in which it is known. What differs, Boër and Lycan propose, is not the semantic content of what is believed but merely the pragmatics of terms used to express it. What I know in knowing *de se* that I am making a mess and what you know in knowing *de re* of me that I am making a mess are precisely the same thing, though known in a different *belief state* or expressed in language with different pragmatics.

On either approach, omniscience might again seem possible. God might, after all, know precisely what I know in knowing that I am making a mess. What he couldn’t do is express it in language with the same pragmatics (Boer and Lycan), or know it in the same *belief state* (Perry).

Neither approach, however, ultimately seems very promising.

When I suddenly realize that the man in the mirror is *me* there is clearly something that I’ve learned. There is something that I didn’t know before that I do know now. There is indeed a piece of crucial information that I’ve just acquired that I didn’t have before—the fact that it is *me* that is making a mess. But the simple fact that there is something learned, or recognized, or realized in such a case is a simple fact to which neither Perry’s nor Boer and Lycan’s accounts can do justice, since on both accounts it is emphatically *not* the case that there would be anything new to learn. Perry’s analysis of such a case would call for a change in *belief state*—But Perry is careful to distinguish *belief states* from what is believed, and thus can’t do justice to the fact that some new piece of information is acquired. Boer and Lycan’s analysis attempts to explain away differences between *de se* and *de re* in terms of pragmatics alone. But they carefully distinguish the pragmatics of expression from the semantic content of what is believed, and thus can’t do
justice to the fact that there is something that comes to be believed in such a case that wasn’t believed before.

Similar comments apply to other indexicals. When I suddenly realize not only that the meeting starts at noon, but that it starts now, there is something that I realize for the first time. When I merely knew that the bomb was going to fall at a spot marked on the map there was a crucial piece of information I lacked but didn’t yet realize that it was going to fall here.

We can also appeal to the fact that what I know carries over to other propositional attitudes. What I come to realize in the mess-making case is precisely the thing that I am then ashamed of: the fact that it is me that is making a mess. For a case without feelings we can note that what I come to realize in realizing it is me who is wearing the blue tie is precisely one of those facts about my own apparel to which I am so indifferent. What I know in knowing that the meeting is starting now is precisely the thing that surprises me that the meeting is starting now. And what I come to know about the bomb fall is precisely what I was afraid that it would fall here.

The straightforward lesson seems to be that it is not merely a belief state that changes when I come to recognize, for example, that it is me that is making a mess. Nor is the difference to be written off as pragmatics of expression. At least part of what has changed is what I know. In the straightforward ontology of what is known, then the straightforward sense of what I know the argument from indexicals does show quite explicitly that no being can know what I know. In defining omniscience we would expect knowledge to be used in the familiar sense. But in that familiar sense there can be no being that knows everything. There can be no omniscient being.
3. The Strengthened Divine Liar

Consider the following:

4. God doesn’t believe that (4) is true.

Is that true or false? If it’s true, God doesn’t believe it and thus cannot be said to know all truths. If it’s false, on the other hand, it’s false that God doesn’t believe it. It must then be true that he believes it. God therefore believes a falsehood. If omniscience is not defined to exclude belief in falsehoods, it clearly should be.

(4) is a strengthened version of the Divine Liar. Using Most Perfect Being Abbruzzese considers merely an unstrengthened version:

5. The MPB believes that (5) is false.

With that version in mind, Abbruzzese seems to think the Divine Liar is particularly easy to dispose of:

Grim’s use of the Liar paradox to impugn the coherence of omniscience is an illegitimate challenge to the coherence of omniscience simply because it is logically impossible to answer the question whether [5] is true or false, since [5] does not express anything at all; it merely contradicts itself. Indeed, statement [5] is, as Swinburne would say, no more
than a garble of words. . . (29)

There is some fairly basic confusion here; Abbruzzese seems to think that contradictions fail to express anything at all, which leaves one wondering what it is that makes them contradictions. That problem aside, however, it is clear that this reasoning does not escape the Strengthened Divine Liar as it appears in (4) above. If (4) fails to express anything at all, or is merely a garble of words, then it clearly is not true. Since Abbruzzese holds that God believes only truths, he must conclude that:

God does not believe that (4) is true.

Simple observation, however, makes it clear that this is (4).

It thus appears that the Strengthened Divine Liar follows quite directly from Abbruzzese's own reasoning. He must therefore be committed to its truth. But if (4) is true, God does not believe it and thus cannot be said to know all truths.

We can construct a sentence like this for any being whatsoever. There can therefore be no omniscient being.

4. Omniscience and the Knower

One argument which Abbruzzese does not address is that which builds on the Paradox of the Knower.
Consider any formal system with axioms adequate for arithmetic. Those axioms formalize statements as simple as the principle that zero is the successor of no natural number. We take them to be true on their face, and moreover true because we take arithmetic to be true.

For any such system, we can encode formulae as numbers; here we will use A to refer to the numbered encoding for a formula A. It is well known that for any such system we will be able to define a derivability relation I such that I(A,B) just in case A is derivable from B.

Let us also introduce a symbol \( \Phi = \) within such a system, applicable in the same way to numerical encodings A for formulae A. We might introduce \( \Phi = \) as a way of representing universal knowledge, for example, the knowledge of an omniscient being within at least this limited formal system. Given any such symbol with any such use we would clearly want to maintain each of the following:

- If something is known by such a being, it is so. \( \Phi(A) \Rightarrow A \)
- This itself is known by such a being. \( \Phi(\Phi(A) \Rightarrow A) \)
- If A is derivable from B in our system, and A is known by such a being, B is known by such a being as well \( I(A,B) \Rightarrow \Phi(A) \Rightarrow \Phi(B) \)

The simple truth, however, is that no symbol can consistently mean what we have proposed \( \Phi = \) to mean, even in a context as limited as the formal system at issue. The addition of these axioms simply renders the system as a whole inconsistent. One way to put the point is
this: that omniscience proves inconsistent in any world adequate for arithmetic.

Both the Knower and the Strengthened Divine Liar, it should be noted, can be finessed by familiar hierarchical techniques. The proposal in Russell, Tarski, Kripke and others is that truth and related predicates form a hierarchy of different levels, each of which applies only to statements (including statements involving truth) on lower levels. On such an approach we know precisely what goes wrong with the Strengthened Divine Liar and the \( \phi \\) predicate: each attempts to apply a truth-related predicate beyond its hierarchically regulated reach.

The formal charms of a hierarchical approach are many. One thing such an approach doesn’t offer, however, is any hope for omniscience. If truth forms such a hierarchy, there can be no notion that applies to truths on all levels. That is precisely what the notion of an omniscient being would require.

5. Second Thoughts on the Cantorian Argument

One thing seems to be established: There can be no set of all truths.

For suppose any set of truths \( T \), and consider its power set \( \mathcal{P}(T) \). For each element of the power set there will be a unique truth \( B \) at least the truth that that element contains a particular truth \( T_1 \) as a member, for example, or that it does not contain \( T_1 \) as a member. By Cantor’s Theorem we know that the power set of any set is larger than the set itself. There will then be more truths than are contained in \( T \). But \( T \) was any set of truths. There can therefore be no set of all truths.

The argument against omniscience requires one further step. Were there an omniscient
being, what such a being knew would constitute a set of all truths. There can be no set of all truths, and therefore can be no omniscient being.

Abbruzzese offers several critiques of the Cantorian argument.

A. Completed Totalities

Grim . . . [assumes] that the multiplicity of truths, $T$, is a completed totality. This, however, need not be so, for the multiplicity of all truths may be, and in fact seems to be, infinite. . . . Indeed, the totality of all truths, I think, must be infinite because, as Grim argues, all truths cannot compose one set, for one can always add another truth to the multiplicity even if only by self-referential propositions, e.g. $\forall T_1$ is true $\land A$ $\forall T_1$ is true $\land A \land$ $\forall T_1$ is true $\land A \land \ldots$ ad infinitum. (32, his punctuation.)

Here again there are some elementary confusions. The argument against a set of all truths holds, precisely as Cantor's Theorem holds, regarding an infinite set of any suggested cardinality. It is indeed clear that there are infinite sets of distinct truths, and it is easy to conjure up sets of truths of higher cardinality than the countable set Abbruzzese exhibits. But no matter what infinite cardinality one proposes for a set of all truths, there will be a truth for each element of its power set and thus will be more truths than it contains. Appeal to infinity does nothing to guard against the argument against a set of all truths.

In the passage quoted Abbruzzese also restricts $\exists$ completed totalities to finite sets, which
is simply a mistake. There are plenty of completed totalities in Cantor’s sense which are fully infinite. But what of the suggestion that the real target of the argument is not all truths in their plurality but merely the notion that they can somehow be thought of as a completed totality as a single set? Much the same suggestion, without some of the confusions, appears also in pieces by Richard Cartwright, D. A. Martin, and Keith Simmons.

Although tempting, this kind of appeal to truth as a many rather than a one turns out to be ineffective against Cantorian argument. The argument does not in the end depend in any essential way on reference to a single set, class, or collection of all truths. It can be phrased directly in terms of the many entirely in the plural, and using only a notion of relations between things.

For properties $P_1$ and $P_2$, the formal properties of relations we require can be outlined as follows. Those things which are $P_1$ can be mapped one-to-one into those things which are $P_2$ just in case there is a relation $R$ such that:

$$\forall x \exists y (P_1x \land P_1y \land \exists z (P_2z \land Rxz \land Ryz \land x = y))$$

Those things which are $P_1$ can be mapped one-to-one onto those things which are $P_2$ just in case (here we simply add a conjunct):

$$\forall x \exists y (P_1x \land P_1y \land \exists z (P_2z \land Rxz \land Ryz \land x = y))$$

$$\exists y (P_2y \land P_1x \land Rx)$$
It will be true for some $P_1$ and $P_2$ that a mapping \textit{into} is possible but a mapping \textit{onto} is not; relative to the things that are $P_1$ there will be too many things that are $P_2$ to allow a full mapping of those things that are $P_1$ \textit{onto} those things that are $P_2$.

Consider, now, any $\exists$ many truths you like the truths that any particular being knows, for example. Consider also truths about one or more of those truths. Using the notions above, phrased entirely in the plural, it is possible to show that the first truths can be mapped \textit{into} but not \textit{onto} the second truths.\footnote{There are more of the latter. No matter what truths are at issue, therefore, they cannot be \textit{all} the truths. Appeal to a plural $\exists$ many rather than a single set-like $\exists$ one thus offers no escape. It is still true that there can be no omniscient being.} There are more of the latter. No matter what truths are at issue, therefore, they cannot be \textit{all} the truths. Appeal to a plural $\exists$ many rather than a single set-like $\exists$ one thus offers no escape. It is still true that there can be no omniscient being.

B. Alternative Set Theories

There have also been attempts to rescue omniscience from the grips of the Cantorian argument by appeal to alternative set theories. I consider all such attempts to date unsuccessful, simply because the set theories appealed to bring with them other commitments that are radically counter-intuitive with regard to the notions of truth at issue. Gary Mar questions the appeal to Cantor, for example, because Cantor's theorem does not hold in some set theories it does not hold, in particular, in the system NF.\footnote{Contrary to Mar's representation, however, the argument need not pause mid-stride and defer to a set-theoretical theorem. It can be phrased throughout quite simply as a philosophical argument regarding truths. The proposal that we shift to a system such as NF, therefore, is really the proposal that we abandon some of the claims about truth that are instrumental in the argument and adopt others, NF-like, instead.}
The technical disappointments of NF are well known. In particular, mathematical induction fails in NF for unstratified conditions. But here it is more relevant that the reason why Cantor’s theorem fails in NF is because the notion of sets is restricted in such a way that only stratified conditions guarantee the existence of corresponding sets. That doesn’t mean that other conditions are in any way illicit or ungrammatical in NF, or in any way fail to apply to the things they appear to apply to. All it means is that we are prohibited from referring to those things to which such conditions apply in terms of sets. If we are to carry this over into the context of a fully philosophical argument concerning truths, however an argument that as we have seen need not be phrased in terms of sets at all the corresponding principle we would have to embrace is that there will be certain pluralities about which there are no truths. In the philosophical context, an NF-like solution would have us cheerfully admit that there are certain things precisely those things that satisfy the diagonal condition of the argument, for example but would force us to conclude that there can nonetheless be no truth about precisely those things. The things themselves fully exist, on such an approach, and they are indeed precisely those things that satisfy the relevant condition. It just that there is no truth about them no truth whatsoever.

As a purely formal system, NF can be regarded as an interesting experiment in set-theoretical axiomatics. The claim that there really are things about which there are no truths, on the other hand, is much more serious. The latter is a fully philosophical claim, and a claim that violates our basic concept of truth. In a philosophical context, it is that claim that any NF-like defense of omniscience against the philosophical argument would demand. For that reason, I think, any NF-like defense must prove philosophically unacceptable. It is of course true that we can block some of the problems at issue by radically reconstituting our notion of truth. In terms
of the notion of truth that we in fact use, however, and in terms of any notion of omniscience defined in terms of that notion of truth, the problems remain.

C. Is the Cantorian Argument Self-Defeating?

I come at last to a difficulty with the Cantorian argument that seems significantly more serious. Although already anticipated in *The Incomplete Universe*, the objection appears forcefully in Plantinga and Grim, *op. cit.*, and is repeated in Abbruzzese:

Aside from these points, however, there is a more important logical flaw in Grim’s Cantorian argument: it is self-reflexively inconsistent. The conclusion that there can be no set of all truths implicitly denies the existence of a set of all propositions, and by denying the existence of such universal propositions, Grim is denying existence to the very conclusion of his argument, a conclusion that, to be meaningful, requires quantification over the totality of all things.

As noted, Abbruzzese uses $\exists \text{MPB} \forall x \exists \text{Most Perfect Being}$

Consider now Grim’s original argument. His conclusion that $\exists \text{there is no set of all truths}$ is equivalent to the universal proposition $\exists \forall x$, if $x$ is a set, then $x$ is not a set of all truths. Moreover, his ultimate conclusion that the MPB does not exist is equivalent to the universal proposition $\exists \forall x$, if $x$ exists, then $x$ is not the MPB. But these propositions
involve universal quantification: the former over sets and truths, the latter over the totality of all things. If Grim’s conclusion is true, and hence denies existence to these types of propositions, then by that very conclusion, it cannot be meaningfully expressed. Thus, it seems to me that Grim’s final objection to omniscience is unsound. (32)

The problem can be tightened. Parallel to the Cantorian argument against a set of all truths is a Cantorian argument to the effect that there can be no proposition about all propositions. But the immediate way of attempting to symbolize that claim, using $\forall x \forall y$ for $x$ is a proposition and $\forall y \forall x$ for $x$ is about $y$ would appear to be:

$$??x(Px \land ??y(Py \land Axy))$$

or equivalently:

$$?x(Px \land ??y(Py \land Axy))$$

or

$$?x(Px \land y(Py \land ?Axy))$$

But such a conclusion has all the marks of a quantification over all propositions about all propositions. If there can be no propositions about all propositions, there can be no
proposition which expresses such a conclusion. If the Cantorian argument at issue were sound, in other words, it could have no true proposition about all propositions as its conclusion. As an argument for any of the conclusions above, therefore, the Cantorian argument cannot be sound. We have reason to be equally suspicious of parallel arguments, one might suggest, including the original Cantorian argument against a set of all truths.

This is indeed an intriguing logical case. It seems to me that there are two obvious responses, each of which breaks some new logical ground but neither of which ultimately affords any consolation for omniscience.

The first response is simply to reject the attempt to phrase the argument conclusion in terms of universal quantification over propositions. The argument is, after all, a reductio, demonstrating that the assumption of a proposition genuinely about all propositions leads to contradiction. We might be forgiven for having thought that a reductio from a premise \( R \) allows us in all cases to conclude straightforwardly that \( \neg R \). This will not be the case, however, where the reductio turns on the inconsistency of a concept which appears in both \( R \) and its negation. In the present case, interestingly enough, the target of the argument of propositional quantification in general is of such wide scope that the principle usually applicable to reductios cannot be applied.\(^{xv} \)

If someone proposes that a particular proposition \( P \) is genuinely about all propositions, however, we will still be able to force him to contradiction by the moves of the argument outlined. We can even see that the argument has a schematic form which will allow us to repeat the argument against the next candidate \( P? \) offered as a proposition about all propositions. What we can\( \neg \) do, interestingly enough, short of falling victim to our own argument, is to draw as
conclusion some universal proposition about all propositions.

There may be other ways in which we can express what it is that the argument shows, however. What it shows is that a particular notion of propositional

> aboutness

if blown to extremes

the notion of a proposition about all propositions

leads directly to contradiction. Any such notion proves inconsistent, as does the notion of omniscience along with it. Here we do

have to be careful to express conclusions about such a notion without ourselves using it, but in cases of existential rather than universal quantification such a need for careful expression has long been clear. Ponce de Leon sought the fountain of youth. But we quite rightly resist the temptation to represent that as the claim that

\[ ?x(Fx \land Spx) \].

Mythical beasts abound in the pages of Revelation. But we must be wary of representing that as an existential quantification over beasts.

The force of the Cantorian argument, it seems to me, stands unimpugned. The interesting logical point is merely that its conclusion cannot be represented in the manner we might first attempt.

Suppose, however, that the case were worse than this. Suppose, purely for the sake of argument, that it could be shown that there was no way to draw a general conclusion from such an argument. I don’t believe that is so. But even if it were, it would provide no consolation for the defender of omniscience. It would still be true, given the next claim P proposed as about all propositions, about all truths, or asserting omniscience, that we could show P to lead quite
The Cantorian argument could be used not to generate some general conclusion, in other words, but merely as a `logic bomb` applied case by case to demolish the next omniscience claim that comes down the line. Even if it in fact afforded us no general positive conclusion, in other words, the Cantorian argument would remain a decisive and devastating tool against claims for omniscience.

6. Conclusion

None of Abbruzzese’s defenses of omniscience seem adequate to save it.

Rightly understood, the core of the essential indexicals argument has nothing to do with feelings. The argument stands that others cannot know what I know in knowing that I am making a mess, for example, and thus stands as an argument that omniscience is impossible.

In a strengthened form, the Divine Liar still seems to show omniscience to be impossible. Abbruzzese’s claim that it expresses nothing at all forces him to draw the Strengthened Divine Liar itself as a conclusion.

The negative conclusion regarding omniscience is further underscored by the reasoning of the Knower.

Of all of Abbruzzese’s attacks, the critiques of the Cantorian argument are the most interesting, and the question of whether the argument is self-defeating is the most interesting of those. It does indeed appear that one cannot represent the conclusion of that argument as a proposition regarding the non-existence of any proposition about all propositions. But that doesn’t mean that the conclusion might not be represented in plenty of other ways. Even if it
couldn’t, moreover even if the Cantorian argument allowed us no positive conclusion the argument structure would remain within reach to use as a logic bomb against the next omniscience claim that comes down the line.

There is no hope yet for the being that knew too much. xvii

Notes


iv. Here I don’t address Castañeda’s proposal in Omniscience and Indexical Reference, Journal of Philosophy 64 (1967), 203-210, which relies on the roundly rejected principle P: that if X knows that Y knows that . . . , X knows what is expressed in situ by the phrase . . . . Castañeda had at any rate given up such an approach, and any associated defense of omniscience, by the
time of Perry. The Problem of the Essential Indexical, @p. cit. For further discussion of Castañeda with regard to omniscience see Grim, Against Omniscience: The Case From Essential Indexicals, @Noûs 29 (1985), 151-180.


vi. The Swinburne approach that Abbruzzese quotes with approval is a propositionalist response to the Liar. But difficulties for propositionalist approaches also continue well beyond this point. See particularly The Incomplete Universe, chapter 1, section 6.


xii. Quine, *Set Theory and its Logic*, Harvard University Press, 1969. J. B. Rosser and Hao Wang showed that no interpretation of $\exists$ compatible with the axioms of NF could make well-orderings of both the lesser-to-greater relation among ordinals and that among finite cardinals except by interpreting $\supseteq$ as something other than identity (Non-standard Models for Formal Logic, @Journal of Symbolic Logic 15 (1950), 113-129). Ernest Specker went on to show that the non-Cantorian sets of NF cause the relations of lesser to greater among cardinals to fail of a well-ordering, entailing the falsehood of the axiom of choice (The Axiom of Choice in Quine = New Foundations for Mathematical Logic, @Proceedings of the National Academy of Sciences 39 (1953), 972-975).

xiii. A consideration of other alternative set theories appears in chapter 4 of *The Incomplete Universe*.


xv. Here I assume classical logic throughout. An intriguing alternative is to borrow elements of the formal structure of intuitionistic logics, which limit the consequences to be drawn from the
negation of a universal quantification.

xvi. In Plantinga and Grim, op. cit, Alvin Plantinga charges that the premises of the Cantorian argument might also fall victim to the same objection: that if the argument were sound, they couldn’t be true. This seems irrelevant from the perspective of the logic bomb. It is the defender of omniscience who buys into the notion of a proposition about all propositions, and he can hardly object to the premises on that grounds that they invoke such a notion. The person who builds the bomb, on the other hand, need himself have no such commitments regarding the coherence of the premises. It is thus only the defender of omniscience who ends up making the universal claim, buying the premises, and facing contradiction and consternation as a result. Plantinga’s other alternative to deny the diagonal seems to me a much better option in this case, but it is not one that Abbruzzese pursues.

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