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A Lottery Paradox for Counterfactuals Without Agglomeration

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Abstract

We will present a new lottery-style paradox on counterfactuals and chance. The upshot will be: combining natural assumptions on (i) the truth values of ordinary counterfactuals, (ii) the conditional chances of possible but non-actual events, (iii) the manner in which (i) and (ii) relate to each other, and (iv) a fragment of the logic of counterfactuals leads to disaster. In contrast with the usual lottery-style paradoxes, logical closure under conjunction—that is, in this case, the rule of Agglomeration of (consequents of) counterfactuals—will not play a role in the derivation and will not be entailed by our premises either. We will sketch four obvious but problematic ways out of the dilemma, and we will end up with a new resolution strategy that is non-obvious but (as we hope) less problematic: contextualism about what counts as a proposition. This proposal will not just save us from the paradox, it will also save each premise in at least some context, and it will be motivated by independent considerations from measure theory and probability theory.

1. A New Paradox

Once a week, a TV lottery takes place which is hosted by a famous entertainer. One day the host has a serious car accident on his way to the studio; out of respect for his condition, the lottery show is being cancelled. At the end of the day, the situation is fairly summarized by our first premise P1:

P1 If A had been the case, B would have been the case.
("If the host had made it to the studio, there would have been the TV lottery that day.")

It happens to be the case that the TV lottery is a lottery with 1.000.000 tickets; let us assume that it would not be the TV lottery anymore if this were not so. And for at least one of the tickets we cannot exclude it to have won if the host had made it to the studio. Taking these together, we have:

P2 Necessarily: B if and only if $C_1 \vee \ldots \vee C_{1000000}$; and there is an i[1], such that the following is not the case: if A had been the case, then C_i would not have been the case.

("Necessarily: the TV lottery would have taken place that day if and only if ticket 1 or ticket 2 or … or ticket 1.000.000 would have won in the TV lottery that day; and there is a ticket i, such that the following is not the case: if the host had made it to the studio, then ticket i would not have won.")

The set of true counterfactuals is of course closed under all logical rules and includes all logical laws. We suppose just a couple of rules to be valid (which are all contained in David Lewis' (1973) logic of conditionals):

P3 All of the following rules are valid[2]:

$\frac{\Box(\varphi\leftrightarrow\psi),\, \varphi\Box\!\!\rightarrow\rho}{\psi\Box\!\!\rightarrow\rho}$ (Left Equivalence), $\frac{\varphi\Box\!\!\rightarrow\psi,\, \Box(\psi\rightarrow\rho)}{\varphi\Box\!\!\rightarrow\rho}$ (Right Weakening), Intersubstitutivity of Necessary Equivalents—of which Left Equivalence is a special case—and

$$\text{Rational Monotonicity:} \frac{\varphi\Box\!\!\rightarrow\rho,\ \neg(\varphi\Box\!\!\rightarrow\neg\psi)}{\varphi\wedge\psi\Box\!\!\rightarrow\rho}$$

What P3 says, is: any of these rules may be applied freely, whether to any of the other premises or in suppositional contexts. Indeed, one may think of the relevant applications as delivering material conditionals that belong to our overall set of premises. In this sense, P3 really constitutes an infinite set of premises. As far as $\Box$ ("necessity") is concerned, we will not need to assume more than what is contained in any so-called normal

1 Since there are only finitely many tickets, here and elsewhere any quantification over i could always be replaced in terms of a long but finite statement of purely propositional form.

2 If $\Box\varphi$ is defined in terms of $\neg\varphi \Box\!\!\rightarrow \bot$, then all these rules follow from David Lewis' axioms and rules. We use this notation: $\rightarrow$ and $\leftrightarrow$ are the material conditional and the material equivalence connectives, respectively. $\Box\!\!\rightarrow$ is the counterfactual conditional connective. Later on we will also use $\Diamond\!\!\rightarrow$ for the conditional-might connective.

system of modal logic; but we will not state this explicitly in terms of a premise.

Note that the following rule has *not* been assumed:

$$\text{Agglomeration:}\ \frac{\varphi \,\Box\!\!\rightarrow \psi,\ \varphi \,\Box\!\!\rightarrow \rho}{\varphi \,\Box\!\!\rightarrow \psi \wedge \rho}$$

And it is not hard to show that Agglomeration does not follow either from Rational Monotonicity together with the very weak rules that had been stated before.[3]

If a counterfactual is true—if φ had been the case, ψ would have been the case—then it is plausible to assume that its consequent ψ should have had a greater chance to have been the case than its negation $\neg\psi$, conditional on the antecedent φ. That is:

> P4 If a counterfactual of the form $\ulcorner$if φ then $\psi\urcorner$ is true, then the conditional chance of ψ given φ is greater than $\frac{1}{2}$.

In fact, in many cases, it should be possible to strengthen P4 by replacing '$\frac{1}{2}$' by some threshold closer to 1 that would be given contextually in some way. If so, P4 above is really not more than just a minimal requirement. P4 is entailed by theories of counterfactuals such as Kvart (1986) and Leitgeb (2012a), and something close to it is also contained in Loewer (2007).

By 'chance' we mean objective, non-epistemic, single-case probability; and, of course, chances as referred to by P4 are to be determined in the actual world, not in some merely possible world. We will speak interchangeably of the chance of a sentence and of the chance of the proposition that is expressed by that sentence.

Since chances are usually taken to be time-relative, too, let us presuppose that the conditional chances in question are taken always at some time shortly before the event that is described by the antecedent φ (assuming that φ does in fact specify an event clearly bounded in time).[4] This has the advantage that at least for all ordinary "common sense" statements φ, if φ is possible at all, then the chance of φ to take place will be greater than 0; hence the conditional chance of ψ given φ will be

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3 However, one can show that Agglomeration is e.g. entailed by Rational Monotonicity together with the stronger rules that are contained in Hawthorne and Makinson (2007). We are grateful to David Makinson for this observation.

4 Actually, both conditional chances and counterfactuals can be assessed relative to different points of time, and determining the time of assessment of a conditional chance statement or a counterfactual to be close to their antecedent time is not generally right; in some contexts, other points of assessment are more appropriate. See section 4.3 of Leitgeb (2012a) for a discussion on this.

well-defined by means of the usual ratio formula for conditional probabilities. In this way we can avoid using anything like Popper functions (on which see Makinson 2011 for an overview): primitive conditional probability measures that would be well-defined and non-trivial even in cases where the antecedent chance is 0. The theory of such extended probability functions is still not accepted widely, and our considerations will be simplified by not having to rely on them. In terms of properties of conditional chance, we will not have to presuppose more than if an ordinary statement φ is possible, then the chances of statements taken conditional on φ can be determined by the usual ratio formula for conditional probabilities, and if φ and ψ are necessarily equivalent, then the conditional chance of a statement given φ is identical to the conditional chance of that statement given ψ. However, we will not state any of these assumptions on chance as extra premises.

Finally, we add one further supposition on our TV lottery and host story: Assume that the host had made it to the studio. Even then there would have been a small chance for the lottery being cancelled: maybe the lottery machine would have been broken; maybe a lunatic would have abducted the TV host from the studio; maybe the lottery show would have been dropped by the boss of the TV channel who had found out that the host had an affair with his wife; or the like. Depending on the empirical circumstances, the chance of cancellation might have been bigger or smaller. Let us assume that the chance for the cancellation to happen was small but not tiny; indeed, we suppose that the chance of the lottery not taking place given the host had made it to the studio is bounded from below by the (presumably, tiny) chance of any particular ticket i winning in this lottery of 1.000.000 tickets given the host had made it to the studio.

Which leads us to our premise P5: Let Case 1 and Case 2 be the following two counterfactual circumstances,

> Case 1: $A \wedge C_i$ ("The host made it to the studio, and ticket i won.")
>
> Case 2: $A \wedge \neg B$ ("The host made it to the studio, but the lottery still did not take place.")

We presuppose that the disjunction of Case 1 and Case 2 describes a possible state of affairs; and we assume that given that state of affairs the proposition $A \wedge C_i$ does not have a chance greater than that of $A \wedge \neg B$:

> P5 For all i: $(A \wedge C_i) \vee (A \wedge \neg B)$ is possible; and the conditional chance of $A \wedge C_i$ given $(A \wedge C_i) \vee (A \wedge \neg B)$ is less than, or equal to, $\frac{1}{2}$.

("For all i: The chance of the host making it to the studio and ticket i winning given that either the host had made it to the studio and ticket i had won or the host had made it to the studio and the lottery had not taken place, is less than, or equal to, one-half. The "given" condition describes a possible state of affairs.")

In case one still worries about this, one might additionally assume the lottery to be fair and, maybe, reformulate our story so that it involves an even greater number of tickets—1.000.000.000 or so. Then P5 should be perfectly harmless.

As things stand, we take it that each of these premises is plausible if considered just by itself. However, one can show that if all of the premises P1–P5 are taken together, they logically imply a contradiction. It is in this sense that the argument from P1–P5 to $\perp$ may be called a paradox.

In section 2 we will demonstrate that the five premises entail a contradiction. Section 3 is devoted to a comparison of the paradox to related ones; as we are going to see, the new paradox differs in structure from all of them. Section 4 deals with the diagnosis of what has gone wrong in the paradox: in particular, we will discuss in detail the options of dismissing one of our premises P1, the second conjunct of P2, P3, and P4; neither of these options will be found particularly attractive. Section 5 presents a new way out of the paradox: a version of contextualism about what counts as a proposition in a context. This proposal will allow us to save each of the five premises in at least some context, and it will be motivated by independent considerations from measure theory and probability theory. Section 6 concludes with an evaluation of the new proposal and its prospects.

2. The Derivation

Let us now turn to the corresponding derivation. First of all, we consider the last conjunct of P2:

> C1 There is an i, such that the following is not the case: if A had been the case, then $\neg C_i$ would have been the case.

In what follows, keep any such i that exists by C1 be fixed—then we have as another intermediate conclusion:

> C1i The following is not the case: if A had been the case, then $\neg C_i$ would have been the case.

So, in the counterfactual situation in question, the winning of that very ticket i would not have been excluded.

With this being in place, using P1, P2, and P3, one can derive a further intermediate conclusion; we will suppress P3 as an explicit premise, instead we simply apply the rules as being permitted by P3 (and, of course, standard propositional logic):

1. $A \Box\!\!\rightarrow B$ (P1)

2. $\Box(B \leftrightarrow C_1 \vee \ldots \vee C_{1000000})$ (P2)

$\underline{3. \neg(A \Box\!\!\rightarrow \neg C_i)(C1i)}$

4. $A \Box\!\!\rightarrow (C_1 \vee \ldots \vee C_{1000000})$ 1., 2. (Right Weakening, $\Box$)

5. $\|$ $\neg\neg(A \Box\!\!\rightarrow \neg(C_i \vee \neg(C_1 \vee \ldots \vee C_{1000000})))$ (Assumption for Reductio)

6. $\|$ $A \Box\!\!\rightarrow \neg(C_i \vee \neg(C_1 \vee \ldots \vee C_{1000000}))$ 5. (Elimination of Double Negation)

7. $\|$ $A \Box\!\!\rightarrow \neg C_i \wedge (C_1 \vee \ldots \vee C_{1000000})$ 6. (Right Weakening, $\Box$)

8. $\|$ $A \Box\!\!\rightarrow \neg C_i$ 7. (Right Weakening, $\Box$)

9. $\|$ $(A \Box\!\!\rightarrow \neg C_i) \wedge \neg(A \Box\!\!\rightarrow \neg C_i)$ 8., 3. (Conjunction)

10. $\neg(A \Box\!\!\rightarrow \neg(C_i \vee \neg(C_1 \vee \ldots \vee C_{1000000})))$ 5.–9. (Reductio)

11. $A \wedge (C_i \vee \neg(C_1 \vee \ldots \vee C_{1000000})) \Box\!\!\rightarrow C_1 \vee \ldots \vee C_{1000000}$ 4., 10. (Rational Monotonicity)

12. $(A \wedge C_i) \vee (A \wedge \neg(C_1 \vee \ldots \vee C_{1000000})) \Box\!\!\rightarrow C_1 \vee \ldots \vee C_{1000000}$ 11. (Left Equivalence)

But 12 implies with premise P4:

> C2 The conditional chance of $C_1 \vee \ldots \vee C_{1000000}$ given $(A \wedge C_i) \vee (A \wedge \neg(C_1 \vee \ldots \vee C_{1000000}))$ is greater than $\frac{1}{2}$.

By P5, P2, and standard modal logic, $(A \wedge C_i) \vee (A \wedge \neg(C_1 \vee \ldots \vee C_{1000000}))$ is possible (and "ordinary"), so we can apply the usual ratio formula for conditional probabilities here. But according to this formula, the conditional chance of $C_1 \vee \ldots \vee C_{1000000}$ given $(A \wedge C_i) \vee (A \wedge \neg(C_1 \vee \ldots \vee C_{1000000}))$ is identical to the conditional chance of $A \wedge C_i$

given $(A \wedge C_i) \vee (A \wedge \neg(C_1 \vee \ldots \vee C_{1000000}))$. And since $(A \wedge C_i) \vee (A \wedge \neg(C_1 \vee \ldots \vee C_{1000000}))$ is necessarily equivalent to $(A \wedge C_i) \vee (A \wedge \neg B)$ by P2 and standard modal logic again, the conditional chance of $A \wedge C_i$ given $(A \wedge C_i) \vee (A \wedge \neg(C_1 \vee \ldots \vee C_{1000000}))$ is in turn equal to the conditional chance of $A \wedge C_i$ given $(A \wedge C_i) \vee (A \wedge \neg B)$. Using this we can derive from C2:

> The conditional chance of $A \wedge C_i$ given $(A \wedge C_i) \vee (A \wedge \neg B)$ is greater than $\frac{1}{2}$.

However, if put together with P5, this leads to a contradiction.

3. Related Arguments

Before we turn to the diagnosis of what has gone wrong here, it is illuminating to compare the new paradox with more familiar ones in order to put it in context and to see where exactly it differs from the others.

Our paradox involves a lottery-type situation. Let us therefore contrast it first with Kyburg's (1961) classical lottery paradox on belief and credence which can be reconstructed so that it proceeds from the following five premises:

> Q1 I am certain that B is the case. ("I am certain that there is one and only one lottery at time t.")
>
> Q2 I am certain that: B if and only if $C_1 \vee \ldots \vee C_{1000000}$. ("I am certain that: there is the lottery at t if and only if ticket 1 wins or ticket 2 wins or … or ticket 1.000.000 wins at t.")
>
> Q3 All standard axioms and rules of doxastic logic are valid.
>
> Q4 If my subjective probability of ψ is greater than 0.9, then I believe that ψ is the case.
>
> Q5 For all i: My subjective probability of C_i is $\frac{1}{1000000}$. ("For all i: My subjective probability of ticket i winning is one over a million.")

In Q1 and Q2, 'certain' means: has subjective probability 1. Q3 makes sure that the agent's set of believed propositions is closed under the usual rules of logical consequence; in particular, if φ is believed, then all of its logical consequences are believed, and if φ and ψ are believed, then so is their conjunction $\varphi \wedge \psi$. For simplicity, let us assume that we are dealing with a perfectly rational agent who always applies deduction competently and who is always perfectly aware of all the conclusions that can be drawn logically from her beliefs. In all this, we take the usual axioms of probability to be implicit in the term 'probability', which is why we won't state them separately.

From premises Q1–Q5 one can derive: I believe that $(C_1 \vee \ldots \vee C_{1000000}) \wedge \neg(C_1 \vee \ldots \vee C_{1000000})$. So the belief system in question ends up inconsistent given the premises.

If compared to Kyburg's famous paradox, our new paradox involves truth (of counterfactuals) where his is about belief, and chance where his deals with credence. And it is crucial to our paradox that we are concerned with conditional notions, not absolute or categorical ones as in Kyburg's paradox. This showed up quite clearly in the last section when we applied a logical rule such as Rational Monotonicity that does not have an "unconditional" counterpart.

This said, it would actually be possible to reinstate our new paradox in terms of *conditional* belief: belief in a proposition under the supposition of another proposition. And the formal resources of a theory such as the (non-probabilistic) theory of belief revision (cf. Gärdenfors 1988) would indeed allow us to carry out the derivation from the last section in these conditional doxastic terms. However, the very notion of conditional belief is more controversial and less common than the one of counterfactual proposition, and the standard Lewisian system of conditional logic is more widely accepted than the rationality postulates for conditional belief as being given by the laws of belief revision; in particular, a rule of inference such as Rational Monotonicity that proceeds from the *absence* of a conditional state is more easily understood if applied to counterfactuals for which this kind of "absence" simply reduces to the falsity of the counterfactual in question. For these reasons, it is preferable for us to express the paradox in counterfactual rather than in doxastic terms.

But the two main differences between Kyburg's and our new paradox lie somewhere else: First of all, where Q4 above is nothing but the *right-to-left* direction of the so-called Lockean thesis (cf. Foley 1993) for a threshold of 0.9, that is, the right-to-left direction of

> ψ is believed by me if and only if my subjective probability of ψ is greater than 0.9,

our new paradox relies on premise P4 which is the analogue of the *left-to-right* direction of the Lockean thesis for a threshold of $\frac{1}{2}$. While adding the left-to-right direction of the Lockean thesis to Q1–Q5 from before allows one to strengthen the "internally" believed inconsistency to a straightforwardly contradictory statement, if taken just by itself the left-to-right direction of the Lockean thesis is perfectly consistent with Q1–Q3 and Q5 above. In contrast, our new paradox involves the true-counterfactual-to-high-conditional-chance version of the left-to-right direction of the Lockean thesis for $\frac{1}{2}$ as its only bridge principle for counterfactuals and chance, and yet a *logical contradiction* follows from it in conjunction with other plausible assumptions.

Secondly, and even more importantly, Kyburg's classical lottery paradox relies on the presence of the closure of rational belief under conjunction. Indeed, famously, according to Kyburg's own diagnosis of his paradox, it is closure under conjunction that ought to be given up (see e.g. Kyburg 1970). However, in our new lottery paradox, the corresponding rule of Agglomeration for conditionals has not been assumed. What we do use instead is Rational Monotonicity, which, as mentioned before, is a rule for conditionals that cannot even be formulated as a closure condition on unconditional belief.[5]

There are other quasi-paradoxical arguments around in the literature on knowledge and chance which do presuppose corresponding knowledge-to-high-chance analogues of the *left-to-right* direction of the Lockean thesis, e.g., in Hawhorne and Lasonen-Aarnio (2009): But in these cases it is typically assumed that some propositions $D_1, \ldots, D_n$ are known, each D_i has a high chance, their conjunction $D_1 \wedge \ldots \wedge D_n$ is also known, but at the same time $D_1 \wedge \ldots \wedge D_n$ is of low chance. The only obvious counterparts of $D_1, \ldots, D_n$ in our paradox are $\neg C_1, \ldots, \neg C_{1000000}$, which in the case of a fair lottery would indeed have high chances; but the counterfactual analogue of knowing each of them—that is, the truth of each counterfactual $A \Box\!\!\rightarrow \neg C_i$—is not validated: in fact, the contrary is the case, since we actually derived C1i: $\neg(A \Box\!\!\rightarrow \neg C_i)$ for a particular i in the last section.

Finally, one can find related paradoxical arguments in the relevant literature that are concerned immediately with conditional chance and counterfactuals, exactly as it is the case in the argument from section 1. Paradigmatically, consider the argument at the beginning of Hawthorne (2005)[6], which can be stated as such:

> R1 If A had been the case, then B would have been the case. ("If I had dropped the plate, it would have fallen to the floor.")
>
> R2 A is possible. ("It could have happened that I dropped the plate.")

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5 We should add that the gist of Kyburg's argument is not actually closure under conjunction *per se* but really any closure condition on rational belief that is at least of the same logical strength as closure under conjunction (modulo some weak background conditions on rational belief that may be defended independently). For instance, closure under conjunction in the lottery paradox could be replaced by closure of rational belief under Modus Ponens (cf. Pagin 1990, Sharon and Spectre, forthcoming): indeed, closure under Modus Ponens entails closure under conjunction given the assumption that every tautology is believed; and closure under conjunction entails closure under Modus Ponens given that belief is closed under one-premise logical consequence, that is, valid inference from one premise. However, Rational Monotonicity is a type of rule that differs from all such closure conditions on unconditional belief. We would like to thank an anonymous referee for urging us to comment on this point.

6 Similar arguments can be found in Hájek, unpublished, and Hawhorne and Lasonen-Aarnio (2009).

R3 The following is not the case jointly: if A had been the case then B would have been the case, and if A had been the case then C might have been the case. ("The following is not the case jointly: if I had dropped the plate it would have fallen to the floor, and if I had dropped the plate it might have flown off sideways.")

R4 If the conditional chance of ψ given φ is greater than 0 (and φ is possible), then $\ulcorner$if φ, then it might be that $\psi\urcorner$ is true.

R5 The conditional chance of C given A is greater than 0. ("The chance of the plate flying off sideways given it had been dropped is greater than 0.")

This set of premises entails a contradiction, and the reasoning is straightforward again.

Once again, we are not interested in evaluating or criticizing this argument, we only want to make clear how it differs from the argument in section 1. Where Hawthorne's R5 is based on quantum-theoretical considerations—for common sense might simply not have regarded R5 to be true—we did not need any particularly scientific assumptions for our own argument.[7] Instead, we did exploit the logic of counterfactuals to a much greater extent than this is the case in Hawthorne's argument. And we did not need to enter any debates on the logical properties of 'might'-counterfactuals, which is clearly an issue in Hawthorne's argument. Indeed, the premises of our argument were spelled out solely in terms of 'would'-counterfactuals and negated 'would'-counterfactuals (as well as statements about possibility, necessity, and chance). We should add that according to David Lewis' (1973) analysis of 'might'-counterfactuals, these are in fact logically equivalent to certain negated 'would'-counterfactuals: but our argument does not rely on this in any way, and we might just as well reject Lewis' analysis of 'might'-counterfactuals.

But most importantly, the reasoning patterns in the two arguments differ substantially, which can be seen clearly if both are reformulated in (roughly) Lewisian semantic terms: While Hawthorne derives a contradiction by locating exceptional $A \wedge \neg B$ circumstances in the closest A-worlds from reasoning from conditional chance statements to 'might'-counterfactuals, we derive a contradiction by partitioning the set of closest A-worlds in terms of $C_1, \ldots, C_{1000000}$: We assume the closest A-worlds to be B-worlds (P1), B to be necessarily equivalent to $C_1 \vee \ldots \vee C_{1000000}$ (first conjunct of P2), and there to be some i, such that some $A \wedge C_i$-worlds are amongst the closest A-worlds (second conjunct of P2). Furthermore, there exist A-worlds (as follows from the first conjunct of P5 as well as from the second

[7] On the other hand, one would probably find an alternative way of formulating Hawthorne's paradox that would not rely on quantum theory in any way.

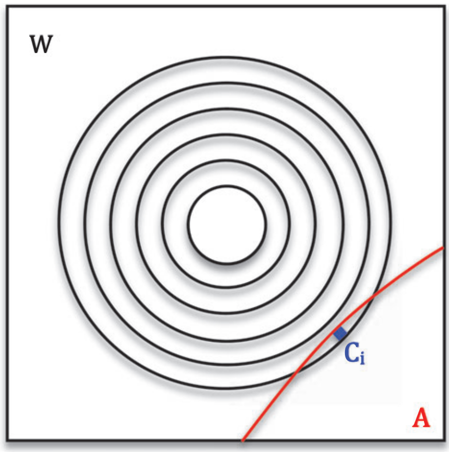

Fig. 1. Comparing the closest $A \wedge C_i$-worlds with $A \wedge \neg B$

conjunct of P2), so P1 is non-vacuously true. By the totality or linearity property of Lewisian sphere systems (which is precisely what is expressed by the validity of the rule of Rational Monotonicity in line with P3), the closest $A \wedge C_i$-worlds must then be closer to the actual world than any of the closest $A \wedge \neg B$-worlds. Therefore, the closest $(A \wedge C_i) \vee (A \wedge \neg B)$-worlds must be $A \wedge C_i$-worlds. Comparing the conditional chance of $A \wedge C_i$ given $(A \wedge C_i) \vee (A \wedge \neg B)$ with that of $A \wedge \neg B$ given $(A \wedge C_i) \vee (A \wedge \neg B)$ (using P4 and P5) finally does the trick. The situation is visualized by figure 1. The formal derivation in the last section captures this pattern of semantic reasoning without relying on any of the rules of Lewis' logic other than the ones mentioned by P3. In a nutshell: For Hawthorne's argument to proceed it suffices to look at the closest A-worlds; but it is crucial to our argument that additionally the closest $(A \wedge C_i) \vee (A \wedge \neg B)$ are being considered.

Note that, given Lewis' (1973) original definition of 'might'-counterfactuals in terms of $\neg(\varphi \Box\!\!\rightarrow \neg\psi)$, and using standard laws of conditional probability, Hawthorne's R4 from above can be reformulated according to the following equivalences (we suppress the possibility statement for φ):

$$Ch(\psi|\varphi) > 0 \rightarrow (\varphi \Diamond\!\!\rightarrow \psi)$$

$$\leftrightarrow \quad Ch(\psi|\varphi) > 0 \rightarrow \neg(\varphi \Box\!\!\rightarrow \neg\psi)$$

$$\leftrightarrow \quad (\varphi \Box\!\!\rightarrow \neg\psi) \rightarrow Ch(\psi|\varphi) = 0$$

$$\leftrightarrow \quad (\varphi \Box\!\!\rightarrow \neg\psi) \rightarrow Ch(\neg\psi|\varphi) = 1$$

In other words, up to logical equivalence and the analysis of 'might' -counterfactuals, Hawthorne's R4 is the principle

$$(\varphi \Box\!\!\rightarrow \psi) \rightarrow Ch(\psi|\varphi) = 1$$

which is but the extreme version of our bridge principle P4. In fact, it would be possible to run Hawthorne's argument based on any *small* threshold ε, where '$Ch(\psi|\varphi) \geq \varepsilon$' would thus replace the initial '$Ch(\psi|\varphi)>0$' statement above, and hence to end up with any large threshold $1-\varepsilon$, where '$Ch(\psi|\varphi)>1-\varepsilon$' would then replace '$Ch(\psi|\varphi)=1$' stated before. With R4 thus revised, that is, up to logical equivalence and the construal of 'might' again

$$(\varphi \Box\!\!\rightarrow \psi) \rightarrow Ch(\psi|\varphi) > 1 - \epsilon,$$

and if in R5 'greater than 0' is replaced by '$\geq \varepsilon$', accordingly, the original pattern of Hawthorne's argumentation would be preserved. For $\epsilon = \frac{1}{2}$, this revised version of R4 would be exactly our premise P4, however this choice of a threshold would then no longer be small enough for Hawthorne's original purposes, since the variant of R5 in which 'greater than 0' got replaced by '$\geq \frac{1}{2}$' would no longer be supported by quantum-theoretical considerations for $\epsilon = \frac{1}{2}$.

We conclude that, in spite of some overlap, Hawthorne's argument remains to differ from the one formulated in section 1 even if an analysis of might-counterfactuals by means of corresponding $\neg(\varphi \Box\!\!\rightarrow \neg\psi)$ statements is presupposed, and in fact even if all might-counterfactuals in Hawthorne's argument had been replaced by corresponding conditionals of the form $\neg(\varphi \Box\!\!\rightarrow \neg\psi)$ from the start.

4. Diagnosis

So what is the problem in the new lottery paradox? At least *prima facie*, there should not be much doubt about the first conjunct of P2 nor about P5 which we can all take to be true quasi-empirical premises about the particular lottery and host in our toy story; and neither of them involves counterfactuals. This leaves us with the obvious options of dropping either of: P1; the second conjunct of P2; P3; P4. We will first state these options briefly and then criticize them:

- Denying P1: This reaction might come in various different brands. One might object to even *formulating* any claim whatsoever that involves counterfactuals, P1 being just of them; the recommendation might be to restrict oneself just to statements on conditional chance when one is dealing with counterfactual possibilities.[8] Or one does

[8] This would be the translation of Richard Jeffrey's rejection of the notion of (qualitative) belief into the present context. Of course, Jeffrey himself would have liked, in addition, to replace statements on objective chance by statements on subjective probability.

not object to counterfactuals *per se*—e.g., counterfactuals with probabilistic consequents might be fine—but one regards all *ordinary* counterfactuals as false; which is what Hájek, unpublished, argues for. Accordingly, by being ordinary, P1 would be false. Or one regards counterfactuals not to be true or false at all, as the Suppositional Theory of Conditionals has it (as held, e.g., by Dorothy Edgington); so P1 would not even be truth-apt, let alone true, though one could still accept P1 in some way other than believing it to be true. Or one takes P1 to be false in view of the additional assumption of P5: for in conjunction with the usual laws of probability (and given that the chance of $(A \wedge C_i) \vee (A \wedge \neg B)$ is greater than 0), P5 entails that the conditional chance of $\neg B$ given A is positive. And maybe a corresponding conditional chance of *not less than* 1 is required for the truth of $A \Box\!\!\rightarrow B$.[9]

- Denying the second conjunct of P2: All of the general worries concerning P1 apply here, too; in the case of the Suppositional Theory of Conditionals, the worries might in fact be greater: for if counterfactuals do not express propositions, it is not clear anymore what it even means to negate them. And the second conjunct of P2 would certainly be unacceptable given Robert Stalnaker's axiom of conditional excluded middle—that is: $(\varphi \Box\!\!\rightarrow \psi) \vee (\varphi \Box\!\!\rightarrow \neg\psi)$—since it would then entail the counterfactual $A \Box\!\!\rightarrow C_i$ which is clearly false according to our story of the unlucky host.

- Denying P3: Here the only salient option would be to drop Rational Monotonicity, as all the other rules are logically very weak and contained in more or less every system of conditional logic in the literature. E.g., Ernest Adams' logic of conditionals, which has been defended by Dorothy Edgington amongst others, does not include Rational Monotonicity as valid; and recently, Lin and Kelly, forthcoming, have proposed a probabilistic explication of conditional belief that does not validate the rule.[10]

9 Leitgeb (2012a) formulates a semantics in which this is the case, even when he argues that his semantics also allows for an interpretation according to which the truth of $A \Box\!\!\rightarrow B$ only requires the conditional chance of B given A to be *close to* 1, as long as 'close to' is understood as a vague term.

10 One should add that in neither of these theories any systematic sense is being made of nested conditionals or of the application of propositional connectives to conditionals. Even just handling negated conditionals, as in the formulation of Rational Monotonicity, is highly problematic in all of these approaches. If $\neg(\varphi \Box\!\!\rightarrow \psi)$ is simply understood as $\varphi \Box\!\!\rightarrow \neg\psi$, as it is sometimes the case in suppositional treatments of conditionals, then Rationality Monotonicity turns out to be valid again even in Adams' logic of conditionals.

- Denying P4: Finally, one might defend the existence of counterfactuals $\ulcorner\varphi$ then $\psi\urcorner$ which are true but where the conditional chance of ψ given φ is less than or equal to $\frac{1}{2}$. That is, with the usual laws of probability: where the conditional chance of $\neg\psi$ given φ is at least as high as the conditional chance of ψ given φ. Although not stated explicitly, such a view is hinted at by Williamson (2009) who argues for the possibility of divergence between, on the one side, a notion of safety that involves counterfactual possibilities (as one feature of knowledge), and sufficiently high objective chance on the other. From a contextualist understanding of counterfactuals as strict implications that are restricted to contextually determined sets of relevant worlds, one might argue against P4 by pointing out that even high-chance sets of worlds might count as irrelevant in certain contexts. From the Lewisian point of view, one might attack P4 for the reason that it runs counter to Lewis' Strong Centering Axiom scheme (which is not included in our premise P3): $\varphi \wedge \psi \rightarrow (\varphi \Box\!\!\rightarrow \psi)$. For let $\top$ be tautology, and let ψ be a low chance truth (assuming that there are such truths). By Strong Centering, $\top \Box\!\!\rightarrow \psi$ is true. But the conditional chance of ψ given $\top$ is just the unconditional chance of ψ, which is low. Thus, P4 would fail in these circumstances.[11] Finally, from a Stalnakerian point of view, if the conditional chance of some ψ given some φ is precisely $\frac{1}{2}$, P4 would seem to contradict Stalnaker's additional axiom scheme of conditional excluded middle again: $(\varphi \Box\!\!\rightarrow \psi) \vee (\varphi \Box\!\!\rightarrow \neg\psi)$.[12]

This is not the place to deal with either of these options in full detail. Instead we will merely point out briefly why we think that each of them is problematic, after which we will move on and propose a new way out of the dilemma raised by the new lottery paradox.

About denying P1: Talking and reasoning in terms of counterfactuals is so deeply entrenched in common sense, philosophy, and maybe even in the applied corners of science that rejecting the whole scale level of counterfactuals should come with too high a price; similarly, an error theory that regards all ordinary counterfactuals to be false would be so revisionary that it should not amount to more than just an ultimate fallback position. And counterfactuals are so close to e.g. disposition ascriptions, which we like to think are true or false, that their

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11 By means of formal models such as the ones that we will introduce in section 5, one can show that Lewis' *Weak* Centering axiom scheme—$(\varphi \Box\!\!\rightarrow \psi) \rightarrow (\varphi \rightarrow \psi)$—is much less problematic in the context of P4.

12 We are very grateful to Timothy Williamson for highlighting these points in a discussion and for urging us to comment on it.

truth-aptness ought not to be sacrificed easily either. For the same reason, it should also be fine to apply propositional connectives to counterfactuals. Finally, the truth of $A \Box\!\!\rightarrow B$ should be consistent with the chance of B given A to be less than 1 by some small real-valued margin, for reasons analogous to those for which my belief in A should be compatible with my subjective probability for A to be less than 1 by some small real-valued margin: for otherwise neither the truth of counterfactual assertions nor that of beliefs would be robust enough to survive the presence of even minor uncertainties which almost inevitably occur in real-world cases.

About denying the second conjunct of P2: More or less the same defense applies as in the case of P1. In addition, Stalnaker's conditional excluded middle is problematic in itself: it is not clear why a negated counterfactual of the form $\neg(\varphi \Box\!\!\rightarrow \psi)$ ought to be logically equivalent to the unnegated counterfactual $\varphi \Box\!\!\rightarrow \neg\psi$, and famously this has been disputed by David Lewis. What the second conjunct of P2 says is just that a certain counterfactual is not true: if the host had made it to the studio, then ticket i would not have won. A particular instance of counterfactual dependency is being denied. But we are not required to interpret this as telling us that any particular ticket would have won.

About denying P3: Rational Monotonicity is logically valid in David Lewis' and in Robert Stalnaker's semantics for counterfactuals, and it would turn out valid, too, if counterfactuals were analyzed as strict conditionals.

As mentioned before, in semantic terms, Rational Monotonicity corresponds to Lewis' similarity or closeness (pre-)orderings[13] $\leq$ being total: for all worlds w,w', it holds that $w \leq w'$ or $w' \leq w$. If totality is dropped, so that overall similarity or closeness is merely assumed to be some partial (pre-)order, then Rational Monotonicity no longer comes out logically valid. Now, let us for the moment disregard the general attractions of total pre-orderings, which are well-known from all the areas in which totality is normally taken as a given, such as decision theory, social choice theory, and belief revision theory; and, say, we also ignore the question of what alternative logical rules for negated counterfactuals ought to replace Rational Monotonicity—for, presumably, there should be *some* rules of inference that are specific for negated counterfactuals. Even then it is still unclear if dropping Rational Monotonicity as a logical rule helps: For even if Rational Monotonicity is not logically, and hence universally, valid, it might still be *locally* truth preserving. In particular: It might simply be a

[13] Formally, Lewisian sphere systems or similarity ordering are *pre*-orders, since anti-symmetry is not presupposed: two numerically distinct worlds may be of equal rank in such an ordering.

feature of the story about our unlucky lottery host that the one application of Rational Monotonicity that was required for the formal derivation in section 2 happens to be truth-preserving. After all, even when a closeness order is not demanded to be total overall, it might still happen to instantiate a pattern of totality *somewhere* in the ordering, if only the (empirical) circumstances are the "right" ones. Simply tell our toy story such that the original transition from lines 4 and 10 to line 11 by means of Rational Monotonicity is accomplished instead by applying Modus Ponens to a new premise of the material conditional form 'line 4 $\wedge$ line 10 $\rightarrow$ line 11': then the same conclusions as before can be drawn without assuming Rational Monotonicity to be logically valid, and it is difficult to see how the (quasi-empirical) truth of that new premise could be ruled out, once the story has been told in the right manner. Indeed, maybe, one might argue for the premise in terms of Lewis-style similarity reasoning again that would apply just to that special case, even when there would be no guarantee that the same type of reasoning could have been applied universally. And if someone argued that this kind of similarity reasoning in favor of 'line 4 $\wedge$ line 10 $\rightarrow$ line 11' would be trumped by reasoning about chances, and that reasoning about chances would speak *against* the truth of this material conditional, then we will see in section 5 that this is not necessarily so: our own solution will preserve at the same time reasoning from some kind of similarity relation *and* from chances without there being any contradiction between them, even though it has to be admitted that the similarity relations that we will employ are unlikely to obey the original Lewisian heuristics (cf. Lewis 1979) of what overall similarity or closeness between worlds supposedly consists in.

About denying P4: Here is how one might want to defend P4 against the attacks mentioned above. On the contextualist point, one should maybe "contextualize" the notion of conditional chance accordingly, by which counterfactuals and conditional chance would be on par again. As far as Strong Centering is concerned, one response would be to say that it is always possible to choose the assessment point of time for chances differently (and so for counterfactuals). If one chose it to be, say, some time after both the antecedent and the consequent time, then if $\varphi \wedge \psi$ is true, the chances of both φ and ψ will be 1 then, and thus the conditional chance of ψ given φ will be 1, too; hence Strong Centering will not cause problems anymore in the presence of P4. In other words: One can have Strong Centering and P4 taken together at least relative to *some* assessment time. Still this would not suffice for Strong Centering to come out as logically valid: But maybe it is not so anyway. Considerations as in Nozick's tracking analysis of knowledge, or on indeterminism (cf. Bennett 2003, section 92), seem to speak against the logical validity of Strong Centering. Also for some true and contingent φ and ψ, one might want $\varphi \mathrel{\Box\!\!\rightarrow} \psi$ to express a counterfactual dependency of ψ on φ, and then to *deny* $\varphi \Box\!\!\rightarrow \psi$ on these grounds, since φ and ψ might "merely" describe some causally and conceptually independent and accidental facts. But that natural move would be ruled out from the start by the logicality of Strong Centering. And if the semantics of $\varphi \Box\!\!\rightarrow \psi$ is to involve some sort of additional ceteris-paribus or normality clause that is to allow for exceptional $\varphi \wedge \neg\psi$-worlds close by the actual world, then one should expect the innermost sphere around the actual world to include worlds other than the actual world, and again Strong Centering would fail. There is one other point which ought to be made about arguments against P4 that are based on considerations on Centering: it is questionable whether they get to the heart of the matter of the paradoxical argument of section 1. After all, the toy story there concerned counterfactual circumstances: circumstances which did not prevail in the actual world. Assume P4 to be adapted only very slightly in the way that an '… and φ is false' clause is added to its antecedent: hence only *proper counter*factuals would be assumed to entail the conditional chance claim that is the consequent of P4. Lewis' Centering axioms would be completely unaffected by P4 thus amended, but the same paradoxical argument could still be run. Finally, concerning the last point of criticism which concerned conditional excluded middle: other than rejecting its logical validity, one might simply change the 'greater than' condition in P4 into a 'greater-than-equals' condition, and replace the 'is less than, or equal to' condition of P5 by 'is less than' in compensation: then once again our argument could proceed as before, the strengthened premise P5 would still be plausible in view of our toy story, and the thus weakened P4 premise would no longer be in conflict with the Stalnakerian principle. Independently, one might hope that some supervaluationist moves would save even the original premise P4 in a Stalnakerian setting.[14]

Over and above defending P4 against these attacks, one might point to some independent reasons for believing it to be true: Say, one regards conditional chance to be nothing but the graded version of counterfactual truth, or counterfactual truth to be nothing but the all-or-nothing version of conditional chance, which is certainly not an implausible view: Then claiming a counterfactual $\ulcorner$if φ then $\psi\urcorner$ to be true and the conditional chance of $\neg\psi$ given φ to be greater than or equal to the conditional chance of ψ given φ should be necessarily false. That is: P5 should be necessarily true. In fact, one should even expect an analogue of the full Lockean thesis to be necessarily satisfied in this case: the truth of a counterfactual should be necessarily equivalent to the corresponding conditional chance being high. Alternatively, if that equivalence is not necessarily the case, the main open question is: what kind of ontic structure is it that the truth condition for counterfactuals is supposed to track? Surely, there must be some answer to the question of

[14] On some of these points, see section 1 of Leitgeb (2012b) for further details.

what it is "out there" in the physical world that counterfactuals are describing and which can be expressed in terms resembling those of the scientists, and if it is not high conditional chance, then finding a good alternative answer constitutes at least an open challenge and a serious worry. Finally, let us focus just on our analogue of the left-to-right direction of the Lockean thesis, that is, P4, and let us assume P4 not to be the case: then how are we to explain that reasoning in terms of counterfactuals seems to be probabilistically reliable? If not a universal claim as in P4, then at least some second-order probabilistic statement ought to hold of the form 'The probability for a counterfactual $\ulcorner\varphi$ then $\psi\urcorner$ to be such that the conditional chance of ψ given φ is high given the counterfactual $\ulcorner\varphi$ then $\psi\urcorner$ is true, is high'.[15] For if reasoning with counterfactuals is not even probabilistically reliable in such a weaker sense, we simply *should not* engage in it at all, because, if only counterfactually, it will lead to falsity in too many cases.

We conclude that none of the four options so far does look particularly attractive. Therefore, the paradox from section 1 should constitute a noteworthy challenge to pretty much everyone who is interested in counterfactuals and chance at all.

5. A New Way Out

Which leads us to a new proposal for how to cope with this paradox: *contextualism about what counts as a proposition*. This proposal will have the advantage of saving, in a sense to be explained and qualified later, *each premise* of the argument in section 1 *in at least some context*. However, there won't be a single context that saves all premises simultaneously, even though P3 (a fragment of the logic of counterfactuals)[16] and P4 (the bridge principle for counterfactuals and conditional chance) will be satisfied in every context. And our proposal will not fall prey to the paradoxical reasoning that led us to inconsistency before.

Of course, contextualist ways out of lottery paradoxes for knowledge and belief have been around for quite some time, but our approach will differ from all of these more standard contextualist solution strategies, and it will do so by relativizing the very notion of proposition to a context.[17] Alterna-

[15] Of course, the interpretation of such second-order probabilities would be in need of serious clarification. Schurz (2001) employs similar second-order probabilistic statements in his explication of the reliability of so-called normic laws in the life sciences, but that is in the context of statistical probability and evolution theory, and even there it is unclear what the appropriate interpretation of the second-order probability measure is meant to be.

[16] In fact we will be able to save much more than just the rules mentioned by P3: we can have all of what David Lewis called the system V of conditional logic if we like.

[17] The only approach in that area of which we know to come close to what we are going to propose is a part of Levi's (1967) theory of acceptance to which we will return in the final section. But Levi's account is itself a non-standard contextualist one.

tively, we might say: it won't be important for us to exploit contextualism in the sense that a counterfactual might express different propositions in different contexts—in analogy with a knowledge ascription that might be taken to have different truth conditions in different contexts—it will only be important whether a counterfactual expresses a proposition in a context at all.

As already mentioned before, there are also contextualist approaches to the semantics for counterfactuals: e.g., recently, Ichikawa (2011) suggested a contextualism about counterfactuals, but that is modeled again after contextualism about knowledge ascriptions; counterfactuals $A \Box\!\!\rightarrow B$ are strict implications which express that all cases satisfy the material conditional $A \supset B$, where the intended range of 'all' is determined by the context. However, once again, Ichikawa's account is not about relativizing the space of propositions to the context, and his argument is also independent of considerations on chance.[18]

Our own proposal is motivated, in the first place, by probabilistic considerations. In probability theory, in any context in which one intends to consider or apply a probability measure, it is common practice to start from some algebra[19] $\mathfrak{A}$ of events or propositions to which probabilities are then assigned. For any given underlying space W (the sample space), every event or proposition in $\mathfrak{A}$ is required to be a subset of W, but not each and every subset of W is necessarily also a member of $\mathfrak{A}$. As measure theorists say: there may be non-measurable sets (that is, non-measurable subsets of W). In fact, in certain circumstances, it *must* be so that non-measurable sets exist, or otherwise some intrinsically plausible postulates on the measure function in question would not be satisfied.

For instance,[20] any proper "geometrical" measure of subsets of the real number line that is supposed to extend the intuitive notion of length of intervals to even complicated sets of real numbers ought to have the following properties: (i) For all bounded intervals $[a,b]$ of real numbers, the measure of such an interval ought to coincide with the length $b-a$ of that interval; (ii) the measure function ought to be invariant under geometrical translations; and (iii) the measure function should satisfy all "logical axioms" that hold for measures in general, such as monotonicity and countable additivity. One can then *prove* that there is no measure function that satisfies all of these assumptions and which at the same time assigns a measure to *every* subset of the real number line. So we find that in at least some

18 See Brogaard and Salerno (2008) for another recent contextualist account of counterfactuals.

19 In fact, usually one starts from a so-called σ-algebra of events which is also closed under taking arbitrary countable unions of events.

20 See any typical textbook on measure theory for the details.

contexts in which a measure space with an infinite sample space W is to be employed, it makes good sense not to require every subset of that sample space to be a member of the algebra $\mathfrak{A}$ of measurable events or propositions. And note that if the intended constraints on the measure had been chosen differently, the class of measurable sets of real numbers might well have been different, too; e.g., if countable additivity is weakened to finite additivity, then there are indeed "geometrical" measures in the sense above which do assign a measure to every set of real numbers. Now, if one thinks of such constraints on measures in the way that one set of constraints might be salient or required in one context but not in another, then the corresponding classes of measurable sets end up being context-dependent as well. This finding may be expected to extend even to cases in which the members of W are not real numbers but where they should rather be interpreted as possible worlds. Of course, the interpretation of measures in measure theory differs substantially from the intended interpretation of the measure to which the premises in our paradox refer—the former are purely mathematical constructions, the latter is supposed to be a function with an "empirical" meaning—but the insight may still carry over in terms of its formal pattern: sometimes it may be necessary not to count every subset of the sample space as belonging to the algebra of events or propositions on which a measure is defined, and it may depend on the context whether a set is counted as event/proposition or whether it is not.

Whilst in the case of probability spaces with a *finite* space W of possible worlds, there is no corresponding mathematical need to omit any of the subsets of W from the algebra in question, it is quite obvious that in almost all, if not *all*, concrete applications of any such probability space, the possible worlds in question are far from "maximally specific" ways the world might be: if anything, they will correspond to more or less coarse-grained partition cells of the class W_{max} of all "maximally specific" ways the world might be.[21] As far as the intended context of application is concerned, it might simply be sufficient to make a distinction between the different partition cells, while it might not be necessary to draw a wedge between any two different members of one and the same partition cell. Or perhaps, for whatever practical limitations, we might not even be able to make more fine-grained distinctions. In any case, once again, from the viewpoint of the "real" class W_{max} of maximally fine-grained possible worlds, an algebra that is based on any such set W of worlds that correspond to partition cells of W_{max} will not

21 We put the ontological question of whether *there are* such maximally specific ways the world might be *at all* to one side here; let us simply assume, for the sake of the argument, possible worlds in this sense do exist. Accordingly, we will disregard the question of whether the class of all metaphysically possible worlds whatsoever (or the class of all physically possible worlds whatsoever) is a proper class or a set.

include each and every proposition, that is, every subclass of the “real” space W_{max} of possible worlds. And again we might take the context to determine the appropriate fineness of grain: in one “coarse-grained” context, various sets of fine-grained worlds will go missing, while in another “fine-grained” context, they may all be present.[22]

Overall, we take this context-dependence of the class of events or propositions to be a stable pattern, and an important insight, from measure theory and probability theory.[23] There are good reasons for thinking so even prior to any considerations concerning the new lottery paradox.

Our next step will be to translate this insight into the domain of counterfactuals.[24] While restricting the algebra of propositions for the chance function will not be important in what follows, restricting the algebra of propositions that can be expressed by counterfactuals will be. In order to show how this might work, we will build a little toy model in which we will be able to evaluate each of the premises of our new lottery paradox relative to contexts. While we will employ the formal structure of a standard Lewis-Stalnaker type semantics for counterfactuals, we do not claim that the usual intended interpretation of this semantics carries over without changes. In particular, the similarity relations between worlds that will be employed below will, presumably, not allow for an interpretation in terms of anything like the Lewisian heuristics for overall similarity or closeness (that is, when determining the kind of similarity required by Lewis: it is of primary importance to minimize violations of laws of nature; it is of secondary importance to…; and so forth). But we take it that this kind of interpretation of similarity between worlds is problematic anyway (without being able to argue for this here; but see section 2.6 of Leitgeb 2012b). For us it will be more important to save premises such as P4, which relate counterfactuals and chance, and which seem plausible independently of—or maybe even in spite of—Lewis' considerations on similarity. At the same time, sticking to the formal structure of Lewis' models will make sure that the logic V of conditionals comes out valid in each and every context, by which our premise P3 will be satisfied as well.

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22 We are grateful to an anonymous referee for emphasizing this point.

23 We should add that some of the problems to do with non-measurable sets can be mitigated by using non-standard probability measures which allow for the assignment of non-standard reals; but there are serious constraints on any such approach which we won't be able to deal with here.

24 Restricting the set of propositions to a proper subalgebra of the full power set algebra of a given set W of possible worlds is *not* a typical move in the possible worlds semantics of modalities. But there are exceptions; see, e.g., Segerberg (2001) who bases his semantics of dynamic doxastic logic on some given topological space of propositions. And a relativization to partitions of the underlying set of worlds is to be found in theories such as Levi's (1967) theory of acceptance and Skyrms' (1984) subjectivist theory of objective chance.

Let us, first of all, assume that every context c in which counterfactuals are to be asserted determines an algebra $\mathfrak{A}_c$ of events or propositions *in c*. If the "sample space" for $\mathfrak{A}_c$ is the class $W_c = W_{max}$ of all possible worlds whatsoever, then not every subclass of W_{max} will be required to be a member of $\mathfrak{A}_c$; and if the sample space is but a set W_c of worlds that correspond to more or less coarse-grained partition cells of W_{max}, then $\mathfrak{A}_c$ will not include each and every proposition—each and every subclass of W_{max}—either.

Let also a Lewisian sphere system $\mathfrak{S}_c$ be determined by each context c which fixes for each world $w \in W_c$ a total similarity or closeness (pre-) ordering $\leq_c^w$ relative to w.[25] We assume that $\mathfrak{A}_c$ and $\mathfrak{S}_c$ are compatible with each other: $\mathfrak{A}_c$ is not just closed under taking complements, unions, intersections (that is, the propositional counterparts of $\neg$, $\vee$, $\wedge$), but also under the propositional counterpart of $\Box\!\!\rightarrow$ as being determined by $\mathfrak{S}_c$ in the usual Lewisian manner. Roughly[26]: For all X, Y in $\mathfrak{A}_c$, there is another proposition, $X \Box\!\!\rightarrow_c Y$,[27] in $\mathfrak{A}_c$, such that for all $w \in W_c$: w is a member of $X \Box\!\!\rightarrow_c Y$ if and only if the set of closest X-worlds relative to w, as being given by $\leq_c^w$, is a subset of Y. For every proposition Z in $\mathfrak{A}_c$, say that Z is true in w if and only if $w \in Z$.

Now suppose a notion of *expressing a proposition in c* to be given in a compositional manner: in particular, a counterfactual $\ulcorner \varphi \Box\!\!\rightarrow \psi \urcorner$ expresses a proposition Z in c if and only if φ expresses a proposition X in c, ψ expresses a proposition Y in c, and $Z = X \Box\!\!\rightarrow_c Y$ (which is a member of $\mathfrak{A}_c$ again). If a sentence does not express a proposition in c, call it *non-entertainable in c*. It is *not* important for our approach that a sentence might express one proposition in one context and a *different* proposition in another context. For us it will only be relevant *whether a sentence expresses a proposition in a context* at all. Indeed, for our purposes, we may well presuppose that if a sentence expresses a proposition Z in a context c, then, if the same sentence also expresses a proposition in another context c', the proposition that it expresses in c' is just Z again.

Define a sentence to be *true in w,c* if and only if the sentence expresses a proposition Z in c, and Z is true in w. If c were a context in which $\mathfrak{A}_c$ happened to be the algebra of all propositions whatsoever, then truth in c would collapse into truth *simpliciter* again (where Lewisian sphere systems would still be determined by contexts). More importantly, if $A_1, \ldots, A_n$ are sentences or formulas in the language of conditional logic (quantifiers being

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25 This part of our proposal is in line with David Lewis' theory which does acknowledge the sensitivity of similarity orderings to conversational contexts.

26 As in all of our previous informal remarks on Lewis' semantics, we will presuppose the so-called limit assumption in order to simplify the Lewisian truth condition for counterfactuals. But nothing will hang on this.

27 In this context, '$\Box\!\!\rightarrow$' does not denote a logical symbol but a logical operation on propositions.

omitted), such that all of them express propositions in c, then, by compositionality, also all of their subformulas express propositions in c. And as long as the logical rules of the system V of conditional logic are applied only to sentences that express propositions in c, all of these rules will preserve truth in w,c for all worlds $w \in W_c$ (whether in categorical or in suppositional contexts), since counterfactuals are still having Lewis-style truth conditions in terms of similarity orderings. Let us express this property of these logical rules by means of: *valid in c*.

Finally, we are ready to reconsider the argument from section 1. We will do so in terms of a little formal toy model that will match the toy story from that section: Let us pretend that the "real" set W_{max} of "maximally fine-grained" possible worlds is the set $\{@, w_1, \ldots, w_{1000000}, w^*\}$. We will consider two contexts c and c': Let the algebra $\mathfrak{A}_c$ include the sets

$$\{@\}, \{w_1, \ldots, w_{1000000}\}, \{w^*\}$$

as well as all sets that result from taking complements, unions, and intersections of these in arbitrary and maybe iterated manner; hence, $\mathfrak{A}_c$ is a set of $8 = 2^3$ propositions. In contrast, let $\mathfrak{A}_{c'}$ be the power set algebra of W_{max}: so $\mathfrak{A}_{c'}$ includes all $2^{1000002}$ subsets of W_{max}. Clearly, c will be a context in which only reasonably unspecific propositions are relevant, whereas c' will allow for "maximally" fine-grained distinctions. Note that, by being the atoms of the algebra $\mathfrak{A}_c$, the sets $\{@\}$,$\{w_1, \ldots, w_{1000000}\}$, $\{w^*\}$ might be said to obtain the role of the more or less coarse-grained possible worlds in the context c. Indeed, we may just as well view $\mathfrak{A}_c$ to be given relative to a set $W_c = \{@, u, w^*\}$ of only *three* worlds and every set in $\mathfrak{A}_c$ to be a corresponding subset of W_c, where the singleton $\{u\}$ takes over the role of the set $\{w_1, \ldots, w_{1000000}\}$. We will switch back and forth between these two ways of viewing $\mathfrak{A}_c$ in what follows. On the other hand, $W_{c'}$ will always remain to be identified with W_{max}.

Now we define a chance measure Ch on the full algebra $\mathfrak{A}_{c'}$ of all subsets of W_{max}. Intuitively, Ch is the chance function of the actual world @, and chances as being given by Ch are meant to be taken at some time shortly before the time of the event described by A, that is, of the host making it to the studio—which, say, is also be the time immediately before the accident is to take place: Let $Ch(\{@\}) = \frac{4}{7}$, so the accident, which does take place in the actual world @, is already very likely to happen; let $Ch(\{w_1\}) = \ldots = Ch(\{w_{1000000}\}) = \frac{2/7}{1000000}$ be the chances of the different tickets to be drawn in the lottery, so that each of the 1.000.000 ticket has the same tiny chance of winning; and let $Ch(\{w^*\}) = \frac{1}{7}$, which will be the small, though not tiny, chance of the host making it to the studio and the lottery still not taking place.[28] If $\mathfrak{A}_c$

[28] Actually, $\frac{1}{7}$ might be a bit too much given our toy story, but never mind.

is considered to be based on $W_c = \{@, u, w^*\}$, then Ch can be regarded to be defined just as well on the propositions in $\mathfrak{A}_c$ by means of the obvious assignment of $Ch(\{u\})$ to be nothing but $Ch(\{w_1, \ldots, w_{1000000}\}) = \frac{2}{7}$.

Next we determine sphere systems $\mathfrak{S}_c$ and $\mathfrak{S}_{c'}$ for the two contexts; for our purposes, it will be sufficient to determine the similarity or closeness orderings $\leq_c^@$ and $\leq_{c'}^@$ only for the actual world @. In the case of c, let[29]

$$@ <_c^@ u <_c^@ w^*$$

and for c', let

$$@ <_{c'}^@ w_1, \ldots, w_{1000000}, w^*$$

This gives us two sphere systems centered on @; the smaller the rank of a world in the ordering, the closer this world is to the actual world @. Note that both orderings $\leq_c^@$ and $\leq_{c'}^@$ satisfy the following salient Compatibility property of total pre-orderings $\leq$ on worlds (or of the strict pre-orderings $<$ of worlds that they determine) with respect to chances:

COMP For all worlds w: the chance of $\{w\}$ is greater than the sum of chances of sets $\{w'\}$ for which $w < w'$.[30]

In the case of c, $\frac{4}{7} > \frac{2}{7} + \frac{1}{7}$, and $\frac{2}{7} > \frac{1}{7}$; and for c': $\frac{4}{7} > 1000000 \cdot \frac{\frac{2}{7}}{1000000} + \frac{1}{7}$. In order for COMP to be satisfied, e.g., we could not have set $w^* <_c^@ u$, nor could we have set $w_i <_{c'}^@ w^*$ for any world w_i, nor $w^* <_{c'}^@ w_1, \ldots, w_{1000000}$, nor $w_i <_{c'}^@ w_j$ for any two worlds w_i, w_j.

Here is a general fact: It is easy to prove that in the case of countably many possible worlds, whenever COMP is satisfied by an ordering $\leq^w$ and by the chance function at w, then if a proposition $X \;\Box\!\!\rightarrow Y$ is true at w, the conditional chance of Y given X at w is greater than $\frac{1}{2}$.[31] In our context, since both $\leq_c^@$ and $\leq_{c'}^@$ satisfy COMP, it follows:

> For all $X, Y \in \mathfrak{A}_c$, for all $w \in W_c$: if $X \Box\!\!\rightarrow_c Y$ is true at w, then $Ch(Y|X) > \frac{1}{2}$.

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29 Commas seperate names for worlds of the same $\leq_c^@$-rank. $<_c^@$ is the strict pre-order that is determined from $\leq_c^@$ by: $w <_c^@ w'$ iff $w \leq_c^@ w'$, but not $w' \leq_c^@ w$. Analogously for c'.

30 In the computer science literature, a similar compatibility condition on probability measures and strict total *orders* (not pre-orders) has been formulated (cf. Snow 1998, Benferhat et al. 1997); so these authors do not allow for ties between worlds. This has the consequence that in their approach only very special probability measures can satisfy COMP, whereas in our approach it is easy to prove that for every probability measure on a countable space there is a total pre-order $\leq$, such that the measure satisfies COMP with respect to that total pre-order.

31 We are suppressing some formal details here which are explained in detail in Leitgeb (unpublished) (though spelled out there for conditional belief contexts).

For all $X, Y \in \mathfrak{A}_{c'}$, for all $w \in W_{c'}$: if $X \Box\!\!\rightarrow_{c'} Y$ is true at w, then $Ch(Y|X) > \frac{1}{2}$.

Therefore, the propositional counterpart of premise P4 from section 1 is satisfied in both contexts.

In order to take the ultimate step to sentences or formulas, assume that in both of our contexts c and c', the sentence A expresses $\{w_1, \ldots, w_{1000000}, w^*\}$, and B expresses $\{w_1, \ldots, w_{1000000}\}$. As far as c is concerned, we might say alternatively: A expresses $\{u, w^*\}$, and B expresses $\{u\}$. In both contexts, it follows that $\neg A$ expresses $\{@\}$, $\neg B$ expresses $\{@, w^*\}$, $A \wedge \neg B$ expresses $\{w^*\}$, and so on. On the other hand, assume only in c' that the sentences of the form C_i express propositions $\{w_i\}$, respectively, whereas each such sentence C_i does not express a proposition in c at all. This manner of determining the expressing relation can be completed in the way that if a sentence expresses a proposition in c and it also expresses a proposition in c', then the propositions in the two cases are identical.

It follows that, e.g., $C_1 \vee \ldots \vee C_{1000000}$ expresses $\{w_1, \ldots, w_{1000000}\}$ in c', $(B \leftrightarrow C_1 \vee \ldots \vee C_{1000000})$ expresses W_{max} in c', and—with an accessibility relation on worlds explained appropriately—$\Box(B \leftrightarrow C_1 \vee \ldots \vee C_{1000000})$ expresses W_{max} in c', too, and precisely the same holds for $\Diamond((A \wedge C_i) \vee (A \wedge \neg B))$; but none of these formulas expresses a proposition in c, by the compositionality of the expressing relation as mentioned before. Thus, c is a context in which only reasonably unspecific sentences, such as B, express propositions, but not specific ones, such as the sentences C_i, which are non-entertainable in c. Perhaps one is interested in c in asserting that $A \Box\!\!\rightarrow B$—if the host had made it to the studio, there would have been the TV lottery that day—but the different possible outcomes of this counterfactual lottery are not being entertained, and indeed not entertain*able*, at all. Accordingly, while $A \Box\!\!\rightarrow B$ follows to be true in @,c—as the unique closest A-world in $\leq_c^{@}$, u, is a B-world—the sentence $A \Box\!\!\rightarrow C_1 \vee \ldots C_{1000000}$ does not express a proposition in c.

What can we say about the truth values of our premises P1–P5 in these contexts c and c' relative to the actual world @ (in which we may suppose our toy story to have taken place)?

Ad c: P1 is true in @,c, as explained. $\Box(B \leftrightarrow C_1 \vee \ldots \vee C_{1000000})$ in P2 is not true in @,c, as pointed out, since it does not express a proposition in c; neither does any formula of the form $\neg(A \Box\!\!\rightarrow \neg C_i)$ again for compositionality reasons. P3 is the case if 'valid' is replaced by 'valid in c', also as explained. P4 follows from $\leq_c^{@}$ satisfying COMP above, once 'true' is replaced by 'true in @,c'. The second conjunct of P5 holds by our definition of Ch, while $\Diamond((A \wedge C_i) \vee (A \wedge \neg B))$ does not express a proposition again in c. Hence, all of our five premises except for P2 and the first conjunct of P5 are satisfied.

Ad c': It turns out that P1 is *not* true in $@,c'$: For w^* is amongst the closest A-worlds as being given by $\leq^{@}_{c'}$, and w^* is a $\neg B$-world. What happens here is that by splitting up u, or $\{w_1, \ldots, w_{1000000}\}$, into little pieces of the form $\{w_i\}$, these worlds w_i cannot count as more similar to the actual world than w^* anymore, or otherwise COMP above would be invalidated, which would thus entail P4 to be invalidated, too. In order for P1 still to hold, it would be necessary that the chance of each proposition $\{w_i\}$ be greater than $\{w^*\}$. In other words: the chance of B given A would have to be much closer to 1 than it is actually—even though it would still not need to be 1 exactly.

All premises other than P1 are true in $@, c'$. In particular, this applies to the first conjunct of P2 which does express a proposition in c', as pointed out before; and the proposition it expresses in c is true in $@$. The same holds for the second conjunct of P2 and the first conjunct of P5.

So every premise of the argument of section 1 is satisfied in some context, although not all of them are satisfied in one and the same context. In fact, one can say more: premises P3 and P4, which are the only general statements amongst the premises, are satisfied in *every* context; and in the case of the other three premises, or of statements like them, there might well be *many* contexts in which they are true. In particular, typical counterfactuals such as P1 may be expected to hold in the (coarse-grained, everyday) contexts in which they are typically asserted. No paradoxical conclusion follows from this, as promised.

More generally, the proposal is then: When we assert ordinary counterfactuals, we normally do so in contexts that determine coarse-grained spaces of propositions, since we are normally not interested in making fine distinctions or in considering very special circumstances. This allows us to reason jointly from general principles such as P3 (conditional logic) and P4 (counterfactual-chance bridge principle) and from ordinary counterfactuals, such as P1 (which, e.g., turns out to be true in the coarse-grained context c). When the context changes with the interests of the subject(s) involved, so that the corresponding space of proposition becomes much more fine-grained—contemplating special outcomes of events, such as the fate of one particular ticket in a lottery, for example—then the general principles remain to be true, but some ordinary counterfactuals may turn out to be false in such a context, unless the conditional chances of their consequents given their antecedents are *very close* to 1. This is exactly what happens to P1 in the fine-grained context c'. In a nutshell: very fine-grained spaces of propositions may demand for true counterfactuals to determine corresponding conditional chances that are very near to 1.

In a sense, something like these results could also have been achieved by merely varying, with the context, which proposition gets expressed by a sentence at all, but where at the same time the underlying algebra of propositions would always taken to be the power set algebra of W_{max}: E.g., as long as one makes sure that only sufficiently coarse-grained propositions are expressed in c by all sentences of the given language $\mathcal{L}$, our previous coarse-grained algebra $\mathfrak{A}_c$ from above could simply be identified with the class of propositions *that are expressed by members of* $\mathcal{L}$ in c, and if all of our original premises P1–P5 were restricted to members of $\mathcal{L}$, then all of our previous conclusions would go through as before. In other words: *instead of restricting the space of propositions, one might instead restrict the space of propositions that can be expressed by a sentence*. Of course, none of the propositions that could not be expressed by members of $\mathcal{L}$ would have any interesting semantic role to play. And the *propositional* counterpart of P4 would not be guaranteed to hold anymore then: for instance, if instead of

$$@ <_c^@ u <_c^@ w^*$$

we would have had

$$@ <_{c'}^@ w_1, \ldots, w_{1000000} <_c^@, w^*$$

in c, with each set $\{w_i\}$ counting as a proposition in c, this would not have mattered as far as A is concerned, since all of $w_1, \ldots, w_{1000000}$ satisfy A and hence a sentence such as A could not drive a wedge between any of the worlds w_i. But there would still be counterfactual *propositions* $X \Box\!\!\rightarrow_c Y$ (that is, subsets of W_{max}) for which

if $X \Box\!\!\rightarrow_c Y$ is true at @, then $Ch(Y|X) > \frac{1}{2}$

would be false, such as, e.g. $\{w_1, w^*\} \Box\!\!\rightarrow_c \{w_1\}$. Ultimately, the choice between a contextualism about propositions and a more conventional one concerning what propositions are being expressed by sentences might be a matter of taste, but if one wants to avoid that a paradoxical argument such as the one from section 1 could still be run on a propositional level, then one needs to restrict the space of propositions in a context, not just which propositions are being expressed by sentences in a context. And restricting the algebra of propositions is certainly much more in line with the practice of most of probability theory, for the simple reason that standard mathematical probability theory does not deal with syntactic items at all. So we stick to our contextualism about propositions instead of one about expressed propositions.

In contrast, note that it is not clear at all how something like the results above could have been achieved by merely varying, with the context, which sphere system one is dealing with, without any additional constraints on the space of propositions or on the space of expressed propositions: for instance, with

$$@ <^{@}_{c'} w_1, \ldots, w_{1000000}, <^{@}_{c} w^*$$

again determining the sphere system of @ in c, with some sentence E expressing $\{w_1\}$ in c and with a sentence F expressing $\{w^*\}$ in c, even the counterfactual *sentence* $E \vee F \Box\!\!\rightarrow E$ would become a counterexample to P4.

6. Evaluation and Prospects

We have presented a new lottery paradox for counterfactuals. The premises of this paradox are, at least at first glance, plausible assumptions on counterfactuals and conditional chance. Nevertheless a contradiction can be derived from them. The paradox differs in various ways from existing paradoxes in the same ballpark—in particular, no rule of closure under conjunction is being employed—and denying either of its premises seems to be problematic.

In the last section we presented a new proposal for how to deal with the paradox: relativize the notion of proposition to the context of assertion of counterfactuals. We have seen that there are independent reasons for thinking that whenever probabilities are involved, one should not expect every set of possible worlds to count as a proposition. If this probabilistic insight is translated into the semantics of counterfactuals with sufficient care—in particular, so that a nice logic of conditionals comes out valid (and hence also P3), and our initial bridge principle on the truth of counterfactuals and conditional chance (P4) is satisfied—then for each of our premises there is a context in which the premise expresses a true proposition in that context. In order to show how this can be done we employed the formal framework of a Lewis-Stalnaker type semantics, however, we refrained from interpreting the similarity or closeness relations between worlds in this semantics in terms of anything like the usual Lewisian heuristics.

If we compare this to the solutions that we had sketched back in section 4, we find that the new proposal is doing pretty well. In the coarse-grained context c of the last section, all premises are true with the exception of P2 and P5, in particular, with the exception of the first conjunct of P2,

$$\text{Necessarily: } B \text{ if and only if } C_1 \vee \ldots \vee C_{1000000},$$

and the first conjunct of P5,

$$\text{Possibly: } (A \wedge C_i) \vee (A \wedge \neg B).$$

But in that context, presumably, one is not even interested in expressing either of them, since one is not interested in the fine-grained possibilities C_i. Instead, one is interested in asserting ‘If A had been the case, B would have been the case’ (‘If the host had made it to the studio, there would have been the TV lottery that day’), which is perfectly harmless, as that counterfactual is true in that context (relative to the actual world).

Now what happens if one shifts one's attention to the more fine-grained possibilities, that is, to the possible outcomes of the counterfactual TV lottery? This means that one changes the context to something like c' from the last section: In c', one is maybe interested in expressing the first conjunct of P2 and the first conjunct of P5, and indeed they turn out to be true in c'. But at the same time, while all other premises remain intact, P1 happens to be false:

If A had been the case, B would have been the case.

In section 4, amongst others, we argued that it was implausible to deny P1 on the grounds that the truth of $A \Box\!\!\rightarrow B$ would require the chance of B given A to be exactly 1, and hence that P1 would have to be false by the simultaneous presupposition of the quasi-empirical premise P5. The truth of $A \Box\!\!\rightarrow B$ should in fact be consistent with the chance of B given A to be less than 1 by some small real-valued margin, since otherwise counterfactuals would be far to sensitive to the possible occurrence of exceptional circumstances. Now, this rebuttal of the initial attack against P1 does not apply to what we found to be the case in c': in order for $A \Box\!\!\rightarrow B$ to be true in @, c', and with COMP from the last section still to be satisfied (or P4 would fail), it would be sufficient for the conditional chance of $\neg B$ given A to be positive but super-tiny, that is, less than the chance of every lottery outcome singleton $\{w_i\}$. It is because *that* is not the case that $A \Box\!\!\rightarrow B$ happens to be false in @, c'. In the previous context c this was not an issue because $\{w_i\}$ did not count as a proposition then. This said, of course, if the conditional chance of $\neg B$ given A were in fact less than the chance of every $\{w_i\}$ given A, then something else would have to go in c' (or a contradiction would follow by our original paradoxical argument): in this case, clearly, the second conjunct of P5 would be false then.

But isn't it *still* plausible that P1 should come out as true, even in a context such as c'? This is where the contextualist strategy kicks in: According to the new proposal, the plausibility of P1 arises out of contexts such as c, in which P1 is indeed true. And the plausibility of the first conjuncts of P2 and P5 is due to contexts such as c', in which these conjuncts hold. But one ought to be careful enough not to mix the two contexts and not to mistake the plausibility of one premise in one context for the plausibility of the same premise in a different context.

For some theses, however, it does not matter in which contexts they are being considered; in our case, P3 and P4 are true in whatever context, which is appropriate in view of the universal, and maybe even necessary,[32] character of the two premises. All other premises are specific for the TV

[32] As far as P4 is concerned, this will depend on how closely—conceptually or ontologically—counterfactuals and conditional chance are tied to each other.

lottery and host situation, and hence the context-dependency of their truth values should be less worrying.

This is not to say that contextualization does not come with a price. In particular, there are obvious barriers of inference from one context to the other, or the original paradox could be reinstated. Indeed, as we have seen, it is possible that one counterfactual is true in one context but false in another, one set of worlds counts as a proposition in one context but not in another, and one counterfactual expresses a proposition in one context but not in another; hence, if any of these verdicts were to be transferred from the one context to the other, one would end up with an inconsistent setting again. Which does not mean that no inferences whatsoever are being permitted to lead from one context to the next. For instance, since we take P3 and P4 for granted universally, if all counterfactuals in a set Φ are true in two contexts (and the actual world), then the results of applying some logically valid rules to members of Φ in the one context can be transferred directly to the other context, as schematically the same rules are valid in both of them (by P3); accordingly, the same conclusions can be drawn from them on conditional chance (by P4). Additionally, as mentioned in the last section, our approach is perfectly consistent with no sentence expressing distinct propositions in any two contexts in which it expresses propositions at all. So it is still fine to assume that sentences mean the same in any two contexts in which they mean anything at all. However, the meaning of the schematic principles P3 and P4 still varies from context to context: for both principles need to be restricted to sentences that express propositions in the very context in question.

We have to leave detailed answers to a couple of important open questions to a different occasion; most importantly: Is the new proposal of help also in some other paradoxical arguments in the same ballpark, such as the ones mentioned in section 4? There are reasons to believe the answer might be ‘yes’. In fact, Levi’s (1967) contextualist solution to the classical lottery paradox is strongly reminiscent of our proposal: in his theory of acceptance, Levi suggests to always start from a partition of “relevant answers” to some question of current interest to the agent; translated into the case of the lottery paradox[33], if the partition is *ticket i will win* vs. *ticket i will not win*, then it may be plausible to accept that *ticket i will not win*; but if the partition is *ticket 1 will win* vs. … *ticket 1.000.000 will win*, then it is no longer plausible to accept that *ticket i will not win* (one should rather suspend judgement then). The main difference to our proposal—apart from the fact that Levi is concerned with unconditional acceptance, whereas we are concerned with the truth of counterfactuals—is that he does not commit himself to anything like our P4, as with his acceptance rule it follows that high sub-

[33] Levi (1967), p. 40, does not actually apply this strategy to the lottery case itself but rather to some reasonably similar statistical example.

jective probability is neither necessary nor sufficient for acceptance.[34] It remains to be seen if our contextualist proposal also applies to cases such as Hawthorne's which we also discussed in section 4.

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[34] Section 13 of Lin and Kelly, forthcoming, shows on very general grounds that "question-invariance"—where questions, in their terminology, are just partitions of the underlying set of possible worlds again—is not to be hoped for independently of one's choice of acceptance rule, as long as the acceptance rule satisfies some general rationality requirements. But their requirements are different from the premises used in the present paper.

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